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ANALYTICAL SOLUTIONS

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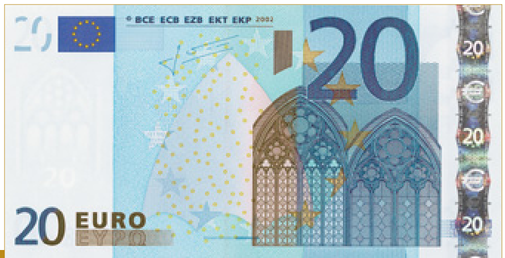

In 2007 all ECB publications feature a motif taken from the €20 banknote.

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Abstract

This paper analyzes the implications of price-setting and incomplete financial markets for optimal monetary cooperation. The main objective is to provide the basic intuitions concerning the role of the main international frictions on optimal policy within a simple Dynamic Stochastic General Equilibrium model. We concentrate on a symmetric two-country DSGE with home bias, incomplete financial markets internationally and imperfect competition together with nominal price rigidities in which the export prices can be denominated either in the producer currency (PCP) or in the consumer currency (LCP). In addition, the model can account both for efficient and inefficient shocks. Our main results are derived in polar cases with efficient steady state and for which the design of the optimal policy is specifically illustrative and can be expressed in terms of targeting rules. In particular, the paper gives some new insights on the optimal exchange rate regime given the structure of shocks and the exchange rate pass-through, as well as on the optimal stabilization of CPI and PPI inflation. We also put into perspective the implication of financial autarky on the optimal management of international spillovers.

Keywords: DSGE models, Optimal monetary policy, New open economy macroeconomics.

JEL classification: E5, F4.

Non-Technical Summary

The main objective of this paper is to analyze simple configurations of a two-country Dynamic Stochastic General Equilibrium (DSGE) model in which the optimal monetary cooperation can be derived easily in order to illustrate the implications of international frictions. This contribution is meant to be helpful for the analysis of optimal monetary cooperation in much more realistic DSGE framework which are able to provide a sensible level a data coherence.

The model shares many features common in open-economy DSGE models. Exchange rate pass-through is incomplete due to some nominal rigidity in the buyer's currency. The specification is flexible enough to continuously link the polar cases of local-currency-pricing (LCP) and producer-currency-pricing (PCP). Financial markets are incomplete internationally and a risk premium on external borrowing is related to the net foreign asset position. Even under flexible prices, purchasing power parity does not hold due to a home bias in aggregate domestic demand. Finally, the economies can be affected by efficient and inefficient shocks.

The main contribution of the paper is to expand on previous studies first by recovering and extending in an open-economy New-Keynesian model most of the results obtained in the New Open Economy Macroeconomics literature, and second by exposing in a unified framework explicit targeting rules for optimal monetary cooperation for three specific configurations of the model parameters. While such targeting rules have already been derived in the PCP case by several authors, the LCP and financial autarky cases constitute, to our knowledge, a novelty within the New Keynesian literature.

Indeed, we restrict our analysis to special cases for which the optimal policy can be derived analytically in terms of targeting rules. In particular, we show that the introduction of "pricing-to-market" changes or complements previous results found in the literature. Under LCP, the monetary authorities should target the consumer price index. A pure CPI inflation targeting strategy implements the optimal outcome when shocks are efficient. An analogous result holds under PCP concerning the optimality of PPI inflation targeting. Moreover, a fixed exchange rate regime may be optimal under LCP in order to alleviate distortions associated with failures of the law of one price. Under PCP, a flexible exchange rate regime is optimal following efficient shocks. However, the presence of cost-push shocks reinforces the case for exchange rate management. Finally, to explore the effect of imperfect risk sharing on optimal cooperation, the case of financial autarky under PCP shows that even with efficient shocks the first best allocation cannot be achieved. This special case also provides some perspective on the

role of the price elasticity of trade with incomplete financial markets, in shaping the optimal response of international relative prices.

Overall, our analysis illustrates the lack of robustness of results about optimal monetary policy in open economies and the importance of correctly modeling international financial market structure and the international price setting.

1 Introduction

The main objective of this paper is to analyze simple configurations of a two-country DSGE model in which the optimal monetary cooperation can be derived easily to illustrate the implications of international frictions.

A large strand of literature aims at analyzing monetary policy in open economies. On the one hand, the so-called ”New Open Economy Macroeconomics” (NOEM) literature, based on the seminal papers of Obstfeld and Rogoff [2000, 1998], examines the conduct of monetary policy in a class of open economy general equilibrium models. This literature focused in particular on the optimality of exchange rate regimes and on the welfare gains from policy coordination. Such topics have been analyzed across a large range of model specifications. It turns out that financial structure, international price setting, preference parameters and nature of shocks are key determinants. Concerning price setting schemes, part of these studies assumes that nominal prices are fixed in the producers' currency, which is called ”producer-currency-pricing” (PCP), so that prices for consumers change one-for-one in the short run with changes in the nominal exchange rate. A number of papers are based however on models in which nominal prices are set in advance in the currency of consumers. In that case, nominal exchange rate changes do not, in the short run, change any prices faced by consumers. It is the ”local-currency-pricing” (LCP) assumption. Within this research agenda, some papers like Devereux and Engel [2003] or Corsetti and Pesenti [2002] focus specifically on the connections between price setting and optimal monetary policy. The hypothesis of complete financial markets is relaxed in several papers like Obstfeld and Rogoff [2002] or Sutherland [2002] in order to analyze the welfare gains from monetary policy coordination. Overall, the recent contribution of Devereux and Engel [2006] which also explore in a unified framework, the implications of price setting and, to a certain extent, imperfect financial markets on optimal monetary policy cooperation, is closely related to the analysis presented in this paper, but from a NOEM perspective basically using a static model and a narrow ”typology” of shocks.

On the other hand, the research program initiated by Rotemberg and Woodford [1997] led to an abundant New Keynesian literature. Whereas, in NOEM models, prices are set on a period by period basis, leading to highly unrealistic dynamics, staggered-price-setting model used in most of this work, has become the workhorse of monetary policy analysis in the closed economy. Thereafter, many studies have extended the analysis to the open economy framework. Indeed, the new generation of dynamic general equilibrium models manages to mix tractability with a rich behavioral structure. The framework we use here is related to those of Benigno and Benigno [2003], Clarida et al. [2002] or Gali and Monacelli [2005] who study optimal monetary policy under PCP and with complete markets. Benigno [2004] introduces "pricing-to-market" in a New Keynesian model but he does not perform welfare analysis nor derive the optimal monetary policy. Smets and Wouters [2002] work on optimal monetary policy in a small open economy under LCP and incomplete financial markets but without using the model consistent welfare approximation.

Our paper belongs to the New Keynesian literature and illustrate the implications of different specifications for the main international economic frictions on the optimal monetary policy cooperation. In particular, we study the impact of price setting and imperfect risk sharing on optimal policy. To gain more intuition on the role of those frictions, we restrict our analysis to special cases for which the optimal policy can be analytically derived. For such specific configurations, we explicitly use an approximation of the welfare function assuming that subsidies are correcting for steady state inefficiencies. Like in the special cases studied by Giannoni and Woodford [2003b] for a closed economy, we derive the targeting rules which implement the optimal monetary policy cooperation.

The model shares many features common in open-economy DSGE models. Exchange rate pass-through is incomplete due to some nominal rigidity in the buyer's currency. The specification is flexible enough to continuously link the polar cases of local-currency-pricing (LCP) and producer-currency-pricing (PCP). Financial markets are incomplete internationally and a risk premium on external borrowing is related to the net foreign asset position. Even under flexible prices, purchasing power parity does not hold due to a home bias in aggregate domestic demand. Finally, the economies can be affected by efficient and inefficient shocks.

The main contribution of the paper is to expand on both strands of literature, first by recovering and extending in an open-economy New Keynesian model most of the NOEM results, and second by exposing in a unified framework explicit targeting rules for optimal monetary cooperation, in the sense of Giannoni and Woodford [2003b], for three specific configurations of the model parameters. While such targeting rules have already been derived in the PCP case by several authors, the LCP and financial autarky cases constitute a novelty, to our knowledge. In addition, the computation of optimal policy in our model is consistent with the Ramsey approach to optimal monetary policy in two-country model which has been studied for example by Faia and Monacelli [2004] under PCP, perfect risk sharing and inefficient steady state. As such, the optimal allocation could be easily derived in our set-up without the restrictive assumptions needed to obtain analytical solutions. Up to a first-order numerical approximation, the same targeting rules could also be derived through the linear-quadratic approximation of the Ramsey problem as in Benigno and Woodford [2006]. While this method presents less of an interest in large DSGE where the intuition about the optimal design of monetary policy is anyway hard to get, it may prove more useful in our simple cases in order to make explicit the policy trade-offs from the second-order approximation of the welfare function. Indeed, the contribution of this paper is also meant to be helpful for the analysis of optimal monetary co-operation in much more realistic DSGE framework which are able to provide a sensible level a data coherence (see for example Adjémian et al. [2007]).

Obviously, we show that the main features of the optimal allocation depends crucially on the price setting schemes and on the type of shocks affecting the economies. When prices are sticky in the producer's currency, we revisit, in a slightly different model, the results of Benigno and Benigno [2006] about the optimal monetary policy and the optimal exchange rate regime. Under specific assumptions and with efficient shocks, pure producer price inflation targeting policies achieve the first best allocation. The nominal exchange rate is thus free to adjust to the required fluctuations of the terms of trade. Nevertheless, with inefficient shocks and financial imperfections, the monetary authorities face additional tradeoffs and the first best allocation cannot be achieved. In that context, exchange rate fluctuations can worsen some policy tradeoffs so that it may be optimal to limit exchange rate movements. We show that, in presence of inefficient shocks, a fixed exchange rate regime is even fully optimal under some parameter restrictions.

These results are not robust to modifications of the price setting assumptions. As previously emphasized in the literature (see Devereux and Engel [2003] for example), the presence of local-currency-pricing, due to the absence of direct exchange rate pass-through, implies that the monetary authorities cannot influence directly the internal terms of trade (see Benigno [2004] for a similar result). Without home bias in national consumption, it can even be shown that terms of trade are independent from monetary policy. Therefore, independently from the shock typology, the monetary authorities cannot manage to completely stabilize the producer inflation rates and the output gaps. Moreover, LCP introduces in the model an additional distortion: with no preference bias, the purchasing-power-parity does not hold and real exchange rate variations induce undesirable volatility in relative consumption. So monetary policies should aim at limiting such movements by targeting directly the consumer price indexes. In particular, we show that under some assumptions, the optimal monetary cooperation under LCP is a "lean against the wind" strategy that adjusts the "consumption gaps" to the consumer-price level fluctuations. The derivation of those targeting rules is one of the main contribution of the paper. Following efficient shocks, it is feasible and optimal to close the consumption gaps and to fully stabilize the consumer-price levels. Furthermore, the predictions about the optimal choice of an exchange rate regime from the PCP case are strongly modified by the LCP assumption. The failure of the law of one price creates new incentives for the monetary authorities to control the exchange rate fluctuations even with efficient shocks.

Finally, in order to explore the effect of imperfect risk sharing on optimal cooperation, we consider the case of financial autarky under PCP. This special case allows us to go beyond Benigno [2001] who did not expose analytical solutions for the optimal policy under imperfect risk sharing and PCP. Such extreme financial market imperfections highlights the associated policy trade-offs: even efficient shocks act as cost-push shocks, pushing inflation rates in opposite directions and preventing to achieve the first best allocation. This special case also provides some perspective on the role of the intratemporal elasticity of substitution with incomplete financial markets in shaping the optimal response of international relative prices.

The rest of the paper is organized as follows. In section 2, the theoretical model is derived. Section 3 presents the simple cases for which the Ramsey problem associated with the optimal monetary cooperation can be formulated to illustrate of the implications of international price setting and international financial frictions in particular. Section 4 concludes.

2 Theoretical model

The world economy is composed of two symmetric countries: Home and Foreign. In each country, there is a continuum of "single-good-firms" indexed on $[0, 1]$, producing differentiated goods that are imperfect substitutes. The number of households is proportional to the number of firms. Consumers receive utility from consumption and disutility from labor. In each country, the consumption baskets aggregating products from both countries have biased preferences towards locally produced goods. Households have identical preferences across countries.

On the labor market, wages are fully flexible. Firms are monopolistic competitors, produce differentiated products and set prices on a staggered basis à la Calvo (1983). Concerning international frictions, we assume that financial markets are complete domestically but incomplete internationally. Moreover export prices are sticky in the producer currency for a fraction of firms and in the buyer currency for the rest. Financial markets are complete domestically but incomplete internationally. In that context, we show that households are identical with respect to their consumption and labor supply choices.

Not only can the economies be affected by efficient shocks (technological shocks). But it is also possible to introduce inefficient shocks that lead to a short run inflation/output gap tradeoff for the conduct of monetary policy. In our model, we might rationalize those shocks as markup fluctuations in the goods market (due to time varying firm-revenue taxes).

For the sake of clarity, most of the derivation will be pursued for country H. Analogous relations hold for country F.

2.1 Consumer's program

At time t, the utility function of a generic domestic consumer h belonging to country H is

$$\mathcal{W}_t(h) = \mathbb{E}_t \left\{ \sum_{j \geq 0} \beta^j \left[\frac{1}{1-\sigma_C} \left(C_{t+j}^h \right)^{1-\sigma_C} - \frac{\tilde{L}}{1+\sigma_L} \left(L_{t+j}^h \right)^{1+\sigma_L} \right] \right\}$$

Households obtain utility from consumption of a distribution good C_t^h, relative to an internal habit depending on past consumption, while receiving disutility from its labor services L_t^h. $\tilde{L}$ is a positive scale parameter.

Financial markets are incomplete internationally. As assumed generally in the literature, Home households can trade two nominal risk-less bonds denominated in the domestic and foreign currency. A risk premium as a function of real holdings of the foreign assets in the entire economy, is introduced on international financing of Home consumption expenditures.

Each household h maximizes its utility function under the following budgetary constraint:

$$\frac{B_{H,t}^h}{P_t R_t} + \frac{S_t B_{F,t}^h}{P_t R_t^* \Psi \left(\frac{S_t \ B_{F,t} - \overline{\overline{B}}_F}{P_t} \right)} + C_t^h = \frac{B_{H,t-1}^h}{P_t} + \frac{S_t B_{F,t-1}^h}{P_t} + \frac{(1-\tau_{W,t}) W_t^h L_t^h + TT_t^h}{P_t} + \frac{\Pi_t^h}{P_t}$$

where W_t^h is the wage, S_t is the nominal exchange rate, TT_t^h are government transfers. Finally, $B_{H,t}^h$ and $B_{F,t}^h$ are the individuals holding of domestic and foreign bonds denominated in local currency. The risk premium function $\Psi(\bullet)$ is differentiable, decreasing and verifies $\Psi(0) = 1$.

Finally, separability of preferences and complete financial markets domestically ensure that households have identical consumption plans.

The first order condition related to consumption expenditures is given by

$$\Lambda_t = C_t^{-\sigma_C} \tag{1}$$

where Λ_t is the lagrange multiplier associated with the budget constraint.

First order conditions corresponding to the quantity of contingent bonds imply that

$$\Lambda_t = R_t \beta \mathbb{E}_t \left[\Lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \tag{2}$$

$$\Lambda_t = R_t^* \Psi \left(\frac{S_t \left(B_{F,t} - \overline{\overline{B}}_F \right)}{P_t} \right) \beta \mathbb{E}_t \left[\Lambda_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]$$

where R_t and R_t^* are one-period-ahead nominal interest rates for country H and F respectively.

The previous equations imply an arbitrage condition on bond prices which corresponds to a modified uncovered interest rate parity (UIP):

$$\frac{R_t}{R_t^* \Psi \left(\frac{S_t \ B_{F,t} - \overline{\overline{B}}_F}{P_t} \right)} = \frac{\mathbb{E}_t \left[\Lambda_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]}{\mathbb{E}_t \left[\Lambda_{t+1} \frac{P_t}{P_{t+1}} \right]} \tag{3}$$

Note that the equivalent arbitrage condition for country F is

$$\frac{R_t^*}{R_t \Psi \left(\frac{B_{H,t}^* - \overline{\overline{B}}_H}{S_t P_t^*} \right)} = \frac{\mathbb{E}_t \left[\Lambda_{t+1}^* \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \right]}{\mathbb{E}_t \left[\Lambda_{t+1}^* \frac{P_t^*}{P_{t+1}^*} \right]} \tag{4}$$

Thereafter, the functional forms used for the risk premium is given by $\Psi(X) = \exp(-2\chi X)$.

2.2 Labor supply and wage setting

In country H, each household is a monopoly supplier of a differentiated labor service. For the sake of simplicity, we assume that he sells his services to a perfectly competitive firm which transforms it into an aggregate labor input using a CES technology $L_t = \left[\int_0^1 L_t(h)^{\frac{1}{\mu_w}} \mathrm{d}h\right]^{\mu_w}$, where $\mu_w = \frac{\theta_w}{\theta_w - 1}$ and $\theta_w > 1$ is the elasticity of substitution between differentiated labor services. The household faces a labor demand curve with constant elasticity of substitution $L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\frac{\mu_w}{\mu_w - 1}} L_t$, where $W_t = \left(\int_0^1 W_t(h)^{\frac{1}{1-\mu_w}} \mathrm{d}h\right)^{1-\mu_w}$ is the aggregate wage rate.

$W_t(h)$ is chosen to maximize the intertemporal utility under the budget constraint and the labor demand for wage setters and the first order condition of this program is:

$$\mu_w \tilde{L} L_t^{\sigma_l} = \Lambda_t w_t \tag{5}$$

where w_t denotes the real wage.

Therefore, the real wage is equal to a markup μ_w over the marginal rate of substitution between consumption and labor.

2.3 Optimal risk sharing

It is worth examining the case of complete asset market structure because our definition of the flexible price equilibrium will assume that financial markets are also complete internationally. In that case, households in both countries are allowed to trade in the contingent one-period nominal bonds denominated in the home currency. This leads to the following risk sharing condition:

$$\frac{\Lambda_t^*}{\Lambda_t} = \kappa RER_t$$

where $RER_t = \frac{S_t P_t^*}{P_t}$ is the real exchange rate and $\kappa = \frac{\Lambda_0^*}{RER_0 \Lambda_0}$ (normalized to 1 given our steady state assumptions). The previous equation is derived from the set of optimality conditions that characterize the optimal allocation of wealth among state-contingent securities.

When markets are complete, it is no use evaluating the current account path in order to determine the relative consumption dynamics. Consumption levels in both countries differ only to the extent that the real exchange rate deviates from purchasing power parity (PPP). In our model, those deviations are allowed for by two assumptions. The first one is the preference bias for locally produced goods, implying that the real exchange rate depends on the terms of trade. The second one is the possibility that prices might not be denominated in the producer currency, which generates failures of the law of one price.

2.4 Distribution sector

A continuum of companies operating under perfect competition mixes local production with imports. There is a home bias in the aggregation, which pins down the degree of openness in the steady state. The distributor technology, $\forall i \in [0,1]$, is given by

$$Y_i = \left[n^{\frac{1}{\xi}} Y_{i,H}^{\frac{\xi-1}{\xi}} + (1-n)^{\frac{1}{\xi}} Y_{i,F}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

$$Y_i^* = \left[(1-n)^{\frac{1}{\xi}} Y_{i,H}^{*\ \frac{\xi-1}{\xi}} + n^{\frac{1}{\xi}} Y_{i,F}^{*\ \frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

where ξ is the elasticity of substitution between bundles Y_H and Y_F. We denote P_H and P_F the price of locally produced goods and imports in country H,and P_F^* and P_H^* the corresponding price indexes for country F.

Cost minimization determines import demands.

$$Y_{H,t} = n \left(T_{H,t}\right)^{-\xi} Y_t, \; Y_{F,t} = (1-n) \left(T_t \, T_{H,t}\right)^{-\xi} Y_t$$
$$Y_{F,t}^* = n \left(T_{F,t}^*\right)^{-\xi} Y_t^*, \; Y_{H,t}^* = (1-n) \left(\frac{T_{F,t}^*}{T_t} \right)^{-\xi} Y_t^*$$

where the consumer prices are defined by

$$P_t = \left[n P_{H,t}^{1-\xi} + (1-n) P_{F,t}^{1-\xi} \right]^{\frac{1}{1-\xi}}$$
$$P_t^* = \left[n P_{F,t}^{*1-\xi} + (1-n) P_{H,t}^{*1-\xi} \right]^{\frac{1}{1-\xi}}$$

$T = \frac{P_F}{P_H}$ and $T^* = \frac{P_F^*}{P_H^*}$ denote the internal terms of trade. We also make use of the relative prices $T_H = \frac{P_H}{P}$ and $T_F^* = \frac{P_F^*}{P^*}$.

2.5 Final goods sector

In country H, final producers for local sales and imports are in perfect competition and aggregate a continuum of differentiated intermediate products from home and foreign intermediate sector. Y_H and Y_F are sub-indexes of the continuum of differentiated goods produced respectively in country H and F. The elementary differentiated goods are imperfect substitutes with elasticity of substitution denoted $\frac{\mu}{\mu-1}$. Final goods are produced with the following technology $Y_H = \left[\int_0^1 Y(h)^{\frac{1}{\mu}} \mathrm{d}h \right]^{\mu}$ and $Y_F = \left[\int_0^1 Y(f)^{\frac{1}{\mu}} \mathrm{d}f \right]^{\mu}$. In the country F, the corresponding indexes are given by $Y_F^* = \left[\int_0^1 Y(f)^{\frac{1}{\mu}} \mathrm{d}f \right]^{\mu}$ and $Y_H^* = \left[\int_0^1 Y(h)^{\frac{1}{\mu}} \mathrm{d}h \right]^{\mu}$. For a domestic product h, we denote $p(h)$ its price on local market and $p^*(h)$ its price on the foreign import market. The domestic-demand-based price indexes associated with imports and local markets in both countries are defined as $P_H = \left[\int_0^1 p(h)^{\frac{1}{1-\mu}} \mathrm{d}h\right]^{1-\mu}$, $P_H^* = \left[\int_0^1 p^*(h)^{\frac{1}{1-\mu}} \mathrm{d}h\right]^{1-\mu}$, $P_F^* = \left[\int_0^1 p^*(f)^{\frac{1}{1-\mu}} \mathrm{d}f\right]^{1-\mu}$ and $P_F = \left[\int_0^1 p(f)^{\frac{1}{1-\mu}} \mathrm{d}f\right]^{1-\mu}$. And domestic demand is allocated across the differentiated goods as follows

$$\begin{cases} \forall h \in [0,1] \quad Y(h) = \left(\frac{p(h)}{P_H}\right)^{-\frac{\mu}{\mu-1}} Y_H, \quad Y^*(h) = \left(\frac{p^*(h)}{P_H^*}\right)^{-\frac{\mu}{\mu-1}} Y_H^* \\ \forall f \in [0,1] \quad Y(f) = \left(\frac{p(f)}{P_F}\right)^{-\frac{\mu}{\mu-1}} Y_F, \quad Y^*(f) = \left(\frac{p^*(f)}{P_F^*}\right)^{-\frac{\mu}{\mu-1}} Y_F^* \end{cases}$$

2.6 Intermediate firms

Intermediate goods are produced with a Cobb-Douglas technology as follows:

$$\begin{cases} \forall h \in [0,1], \quad Y_t(h) = \varepsilon_t^A L_t(h) \\ \forall f \in [0,1], \quad Y_t^*(f) = \varepsilon_t^{A*} L_t^*(f) \end{cases}$$

where ε_t^A and ε_t^{A*} are exogenous technology parameters. Each firm sells its products in the local market and in the foreign market. We denote $Y_H(h)$ and $Y_H^*(h)$ (respectively $Y_F^*(f)$ and $Y_F(f)$) the local and foreign sales of domestic producer h (respectively foreign producer f) and we define $L_H(h)$ and $L_H^*(h)$ (respectively $L_F^*(f)$ and $L_F(f)$) the corresponding labor demand.

Firms are monopolistic competitors and produce differentiated products. For local sales, firms set prices on a staggered basis *à la* Calvo (1983). In each period, a firm h (resp. f) faces a constant probability $1 - \alpha_H$ (resp. $1 - \alpha_F^*$) of being able to re-optimize its nominal price. This probability is independent across firms and time in a same country. The average duration of a rigidity period is $\frac{1}{1-\alpha_H}$ (resp. $\frac{1}{1-\alpha_F^*}$). If a firm cannot re-optimize its price, the price evolves according to the following simple rule:

$$p_t(h) = p_{t-1}(h)$$

As the distribution of prices among the share α_H of producers unable to re-optimize at t is similar to the one at $t-1$, the aggregate price index has the following dynamics:

$$P_{H,t}^{\frac{1}{1-\mu}} = \alpha_H P_{H,t-1}^{\frac{1}{1-\mu}} + (1-\alpha_H)\, \hat{p}_t^{\frac{1}{1-\mu}}(h)$$

The firm h chooses $\hat{p}_t(h)$ to maximize its intertemporal profit

$$\mathbb{E}_t \left[\sum_{j=0}^{\infty} \alpha_H^j \Xi_{t,t+j} \left((1-\tau_{t+j})\hat{p}_t(h) Y_{H,t+j}(h) - MC_{t+j} P_{H,t+j} Y_{H,t+j}(h)\right)\right]$$

where $Y_{H,t+j}(h) = \left(\frac{\hat{p}_t(h)}{P_{H,t}}\right)^{-\frac{\mu}{\mu-1}} \left(\frac{P_{H,t}}{P_{H,t+j}}\right)^{-\frac{\mu}{\mu-1}} Y_{H,t+j}$.

$\Xi_{t,t+j} = \beta^j \frac{\Lambda_{t+j} P_t}{\Lambda_t P_{t+j}}$ is the marginal value of one unit of money to the saver households. MC_{t+j} is the real marginal cost deflated by the producer-price for local sales and τ_t is a time-varying tax on firm's revenue. Due to our assumptions on the labor market, the real marginal cost is identical across producers.

$$MC_t = \frac{w_t}{\varepsilon_t^A T_{H,t}} \tag{6}$$

In our model, all firms that can re-optimize their price at time t choose the same level.

The first order condition associated with the firm's choice of $\hat{p}_t(h)$ is

$$\mathbb{E}_t \left[\sum_{j=0}^{\infty} \alpha_H^j \Xi_{t,t+j} Y_{H,t+j}(h) P_{H,t+j} \left((1-\tau_{t+j}) \frac{\tilde{p}_t(h)}{P_{H,t}} \frac{P_{H,t}}{P_{H,t+j}} - \mu MC_{t+j} \right) \right] = 0$$

This price setting scheme can be written in the following recursive form $\frac{\hat{p}_t(h)}{P_{H,t}} = \mu \frac{Z_{H1,t}}{Z_{H2,t}}$ where

$$\mathcal{Z}_{H1,t} = \Lambda_t MC_t Y_{H,t} T_{H,t} + \alpha_H \beta \mathbb{E}_t \left[\Pi_{H,t+1}^{\frac{\mu}{\mu-1}} \mathcal{Z}_{H1,t+1} \right] \tag{7}$$

and

$$\mathcal{Z}_{H2,t} = (1-\tau_t) \Lambda_t Y_{H,t} T_{H,t} + \alpha_H \beta \mathbb{E}_t \left[\Pi_{H,t+1}^{\frac{1}{\mu-1}} \mathcal{Z}_{H2,t+1} \right] \tag{8}$$

Accordingly, the aggregate price dynamics leads to the following relation.

$$1 = \alpha_H \Pi_{H,t}^{\frac{1}{\mu-1}} + (1-\alpha_H) \left(\mu \frac{\mathcal{Z}_{H1,t}}{\mathcal{Z}_{H2,t}} \right)^{\frac{1}{1-\mu}} \tag{9}$$

When the probability of being able to change prices tends towards unity, this implies that the firm sets its price equal to a markup $\frac{\mu}{(1-\tau_t)}$ over marginal cost. The time varying tax on firms' revenue is affected by an i.i.d shock defined by $1-\tau_t = \left(1-\overline{\overline{\tau}}\right) \varepsilon_t^P$.

Equations analogous hold for foreign producers and governs the dynamics of $\Pi_{F,t}^*$ as follows

$$\mathcal{Z}_{F1,t}^* = \Lambda_t^* MC_t^* Y_{F,t}^* T_{F,t}^* + \alpha_F^* \beta \mathbb{E}_t \left[\Pi_{F,t+1}^{*\frac{\mu}{\mu-1}} \mathcal{Z}_{F1,t+1}^* \right] \tag{10}$$

$$\mathcal{Z}_{F2,t}^* = (1-\tau_t^*) \Lambda_t^* Y_{F,t}^* T_{F,t}^* + \alpha_F^* \beta \mathbb{E}_t \left[\Pi_{F,t+1}^{*\frac{1}{\mu-1}} \mathcal{Z}_{F2,t+1}^* \right] \tag{11}$$

and

$$1 = \alpha_F^* \Pi_{F,t}^{*\frac{1}{\mu-1}} + (1-\alpha_F^*) \left(\mu \frac{\mathcal{Z}_{F1,t}^*}{\mathcal{Z}_{F2,t}^*} \right)^{\frac{1}{1-\mu}} \tag{12}$$

where the real marginal cost for country F is given by,

$$MC_t^* = \frac{w_t^*}{\varepsilon_t^{A*} T_F^*} \tag{13}$$

Similarly, the time varying tax on firms' revenue is affected by an i.i.d shock defined by $1 - \tau_t^* = \left(1 - \overline{\overline{\tau}}\right) \varepsilon_t^{P*}$.

Concerning exports, we assume that, in country H, a fraction η (respectively η^* in country F) of exporters exhibit producer-currency-pricing (PCP) while the remaining firms exhibit local-currency-pricing (LCP). Consequently, aggregate export prices denominated in foreign currency are given by

$$P_H^* = \left[\eta \left(\frac{P_{H,t}}{S_t}\right)^{\frac{1}{1-\mu}} + (1-\eta)\, \tilde{P}_H^{*\frac{1}{1-\mu}}\right]^{1-\mu} \text{ and } P_F = \left[\eta^* \left(S_t P_{F,t}^*\right)^{\frac{1}{1-\mu}} + (1-\eta^*)\, \tilde{P}_F^{\frac{1}{1-\mu}}\right]^{1-\mu}.$$

The aggregate LCP export price indexes are accordingly defined as

$$\tilde{P}_H^* = \left[\frac{1}{1-\eta}\int_\eta^1 p^*(h)^{\frac{1}{1-\mu}} dh\right]^{1-\mu} \text{ and } \tilde{P}_F = \left[\frac{1}{1-\eta^*}\int_{\eta^*}^1 p(f)^{\frac{1}{1-\mu}} df\right]^{1-\mu}.$$

Let us define the following relative prices $R\tilde{E}R_H = \frac{S\tilde{P}_H^*}{P_H}$, $R\tilde{E}R_F = \frac{\tilde{P}_F}{SP_F^*}$ and $\tilde{T} = \frac{\tilde{P}_F}{P_H}$. Export margins relative to local sales are denoted $RER_H = \frac{SP_H^*}{P_H}$ and $RER_F = \frac{P_F}{SP_F^*}$. If there is some form of international price discrimination, those ratios figure the relative profitability of foreign sales compared with the local ones. $RER_t = \frac{S_t P_t^*}{P_t}$ is the real exchange rate measured with the consumer price indexes.

LCP exporters also set their prices on a staggered basis and face the same nominal rigidities as the local producers.

Consequently, the inflation dynamics of LCP export prices for the country H, $\tilde{\Pi}_{H,t}^*$, is described by the following three equations

$$\tilde{\mathcal{Z}}_{H1,t}^* = \Lambda_t MC_t Y_{H,t}^* T_{H,t} + \alpha_F^* \beta \mathbb{E}_t \left[\tilde{\Pi}_{H,t+1}^{*\frac{\mu}{\mu-1}} \tilde{\mathcal{Z}}_{H1,t+1}^*\right] \tag{14}$$

$$\tilde{\mathcal{Z}}_{H2,t}^* = (1-\tau_t)\Lambda_t Y_{H,t}^* T_{H,t} R\tilde{E}R_{H,t} + \alpha_F^* \beta \mathbb{E}_t \left[\tilde{\Pi}_{H,t+1}^{*\frac{1}{\mu-1}} \tilde{\mathcal{Z}}_{H2,t+1}^*\right] \tag{15}$$

$$1 = \alpha_F^* \left(\tilde{\Pi}_{H,t}^*\right)^{\frac{1}{\mu-1}} + (1-\alpha_F^*) \left(\mu \frac{\tilde{\mathcal{Z}}_{H1,t}^*}{\tilde{\mathcal{Z}}_{H2,t}^*}\right)^{\frac{1}{1-\mu}} \tag{16}$$

LCP export price inflation for country F, $\tilde{\Pi}_{F,t}$, is given by the equivalent formulation

$$\tilde{\mathcal{Z}}_{F1,t} = \Lambda_t^* MC_t^* Y_{F,t} T_{F,t}^* + \alpha_H \beta \mathbb{E}_t \left[\tilde{\Pi}_{F,t+1}^{\frac{\mu}{\mu-1}} \tilde{Z}_{F1,t+1} \right] \tag{17}$$

$$\tilde{\mathcal{Z}}_{F2,t} = (1-\tau_t^*) \Lambda_t^* Y_{F,t} T_{F,t}^* R\tilde{E}R_{F,t} + \alpha_H \beta \mathbb{E}_t \left[\tilde{\Pi}_{F,t+1}^{\frac{1}{\mu-1}} \tilde{Z}_{F2,t+1} \right] \tag{18}$$

$$1 = \alpha_H \tilde{\Pi}_{F,t}^{\frac{1}{\mu-1}} + (1-\alpha_H) \left(\mu \frac{\tilde{\mathcal{Z}}_{F1,t}}{\tilde{\mathcal{Z}}_{F2,t}} \right)^{\frac{1}{1-\mu}} \tag{19}$$

2.7 Market clearing conditions

Aggregate domestic demands are given by $Y_t = C_t$ and $Y_t^* = C_t^*$

Aggregate productions verify

$$Z_t = \varepsilon_t^A L_t \tag{20}$$

$$Z_t^* = \varepsilon_t^{A*} L_t^* \tag{21}$$

where L_t and L_t^* are the labor input.

Market clearing conditions in goods markets lead to the following relations

$$Z_t = n \Delta_{H,t} \left(T_{H,t} \right)^{-\xi} C_t + (1-n) \Delta_{H,t}^* \left(\frac{T_{F,t}^*}{T_t^*} \right)^{-\xi} C_t^* \tag{22}$$

$$Z_t^* = n \Delta_{F,t}^* \left(T_{F,t}^* \right)^{-\xi} C_t^* + (1-n) \Delta_{F,t} \left(T_t T_{H,t} \right)^{-\xi} C_t \tag{23}$$

where $\Delta_{H,t} = \int_0^1 \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\frac{\mu}{\mu-1}} \mathrm{d}h$, $\Delta_{H,t}^* = \int_0^1 \left(\frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\frac{\mu}{\mu-1}} \mathrm{d}h$, $\Delta_{F,t}^* = \int_0^1 \left(\frac{p_t^*(f)}{P_{F,t}^*} \right)^{-\frac{\mu}{\mu-1}} \mathrm{d}f$ and $\Delta_{F,t} = \int_0^1 \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\frac{\mu}{\mu-1}} \mathrm{d}f$ measure price dispersions among products of country H and F, sold locally or exported. Those indexes have the following dynamics

$$\Delta_{H,t} = (1-\alpha_H) \left(\mu \frac{\mathcal{Z}_{H1,t}}{\mathcal{Z}_{H2,t}} \right)^{-\frac{\mu}{\mu-1}} + \alpha_H \Delta_{H,t-1} \Pi_{H,t}^{\frac{\mu}{\mu-1}} \tag{24}$$

$$\Delta_{F,t}^* = (1-\alpha_F^*) \left(\mu \frac{\mathcal{Z}_{F1,t}^*}{\mathcal{Z}_{F2,t}^*} \right)^{-\frac{\mu}{\mu-1}} + \alpha_F^* \Delta_{F,t-1}^* \Pi_{F,t}^{*\frac{\mu}{\mu-1}} \tag{25}$$

$$\Delta_{H,t}^* = \eta \Delta_{H,t} + (1-\eta) \tilde{\Delta}_{H,t}^* \tag{26}$$

$$\tilde{\Delta}_{H,t}^* = (1-\alpha_F^*) \left(\mu \frac{\tilde{\mathcal{Z}}_{H1,t}}{\tilde{\mathcal{Z}}_{H2,t}} \right)^{-\frac{\mu}{\mu-1}} + \alpha_F^* \tilde{\Delta}_{H,t-1}^* \tilde{\Pi}_{H,t}^{*\frac{\mu}{\mu-1}} \tag{27}$$

$$\Delta_{F,t} = \eta^* \Delta^*_{F,t} + (1-\eta^*)\,\Delta_{F,t} \tag{28}$$

$$\tilde{\Delta}_{F,t} = (1-\alpha_H)\left(\mu\frac{\tilde{Z}_{H1,t}}{\tilde{Z}_{H2,t}}\right)^{-\frac{\mu}{\mu-1}} + \alpha_H \tilde{\Delta}_{F,t}\tilde{\Pi}_{F,t}^{\frac{\mu}{\mu-1}} \tag{29}$$

Equilibrium in the bond markets implies that $B_{F,t}+B^*_{F,t}=0$ and $B_{H,t}+B^*_{H,t}=0$. Moreover, demand for bonds denominated in currency F emanating from agents in country H is given by

$$\frac{S_t B_{F,t}}{\Psi P_t R^*_t} - \frac{B^*_{H,t}}{P_t R_t} = \frac{S_t B_{F,t-1}}{P_t} - \frac{B^*_{H,t-1}}{P_t} + T_{H,t}Y_{H,t} + RER_t\frac{T^*_{F,t}}{T^*_t}Y^*_{H,t} - Y_t \tag{30}$$

We abstracted here from the risk premium in the accumulation equation for the net foreign assets. Up to a first order approximation, this modification is neutral but at a second order, it brings some symmetry in the effect of financial market imperfections on the stochastic steady state for each country.

Some relative prices have finally to be defined as a function of stationary variables. First, the 4 inflation rates for export prices and local sales prices determine 3 relative prices: 2 relative export margins for LCP producers and internal terms of trade for country H.

$$R\tilde{E}R_{H,t} = R\tilde{E}R_{H,t-1}\frac{\tilde{\Pi}^*_{H,t}\,(1+\Delta S_t)}{\Pi_{H,t}} \tag{31}$$

$$R\tilde{E}R_{F,t} = R\tilde{E}R_{H,t-1}\frac{\tilde{\Pi}_{F,t}}{\Pi^*_{F,t}\,(1+\Delta S_t)} \tag{32}$$

$$T_t = T_{t-1}\frac{\Pi_{F,t}}{\Pi_{H,t}} \tag{33}$$

The following variables are deduced from the previous three relative prices.

$$RER_{H,t} = \left[\eta + (1-\eta)\,R\tilde{E}R_{H,t}^{\frac{1}{1-\mu}}\right]^{1-\mu} \tag{34}$$

$$RER_{F,t} = \left[\eta + (1-\eta)\,R\tilde{E}R_{F,t}^{\frac{1}{1-\mu}}\right]^{1-\mu} \tag{35}$$

$$T^*_t = \frac{T_t}{RER_{H,t}RER_{F,t}} \tag{36}$$

$$T_{H,t} = \left[n_t + (1-n_t)T_t^{1-\xi}\right]^{\frac{1}{\xi-1}} \tag{37}$$

$$T^*_{F,t} = \left[n^*_t + (1-n^*_t)T_t^{*\xi-1}\right]^{\frac{1}{\xi-1}} \tag{38}$$

$$RER_t = RER_{H,t}T_{H,t}\frac{T^*_t}{T^*_{F,t}} \tag{39}$$

Finally, aggregate export price inflation rates and after-tax CPI inflation rates are given by

$$\Pi_{H,t}^* = \frac{RER_{H,t}}{RER_{H,t-1}} \frac{\Pi_{H,t}}{(1+\Delta S_t)} \tag{40}$$

$$\Pi_{F,t} = \frac{RER_{F,t}}{RER_{F,t-1}} \Pi_{F,t}^* \left(1+\Delta S_t\right) \tag{41}$$

$$\Pi_t = \frac{T_{H,t}}{T_{H,t-1}} \Pi_{H,t} \tag{42}$$

$$\Pi_t^* = \frac{T_{F,t}^*}{T_{F,t-1}^*} \Pi_{F,t}^* \tag{43}$$

The aggregate conditional welfare for each country are defined by $\mathcal{W}_{H,t} = \int_0^1 \mathcal{W}_t(h)\mathrm{d}h$ and $\mathcal{W}_{F,t} = \int_0^1 \mathcal{W}_t(f)\mathrm{d}f$.

2.8 Competitive equilibrium

The competitive equilibrium is a set of stationary 18 processes for country H, Z_t, C_t, Λ_t, L_t, MC_t, Π_t, $\Pi_{H,t}^*$, $\Pi_{H,t}$, $\Delta_{H,t}$, $\mathcal{Z}_{H1,t}$, $\mathcal{Z}_{H2,t}$, $\tilde{\Pi}_{H,t}^*$, $\tilde{\Delta}_{H,t}^*$, $\tilde{\mathcal{Z}}_{H1,t}^*$, $\tilde{\mathcal{Z}}_{H2,t}^*$, w_t, $\Delta_{H,t}^*$, $B_{F,t}$ as well as the analogous 18 processes for country F, 9 relative prices $R\tilde{E}R_{H,t}$, $R\tilde{E}R_{F,t}$, $RER_{H,t}$, $RER_{F,t}$, RER_t, T_t, T_t^*, $T_{H,t}$, $T_{F,t}^*$ and the depreciation rate ΔS_t. The 46 stationary processes satisfy the relations (1)-(43) and the analogous of equations (1), (2), (5) for country F, given exogenous stochastic processes for country H, ε_t^A, ε_t^P with the analogous shocks for country F, R_t, R_t^*, initial conditions for country H, $C_{-1}\Delta_{H,-1}$, $\tilde{\Delta}_{H,-1}^*$, $\Pi_{H,-1}$, $\tilde{\Pi}_{H,-1}^*$, w_{-1}, analogous initial conditions for country F, and $R\tilde{E}R_{H,-1}$, $R\tilde{E}R_{F,-1}$, T_{-1}.

2.9 The Ramsey formulation of optimal monetary policy cooperation

As in Schmitt-Grohe and Uribe [2005], we assume that the monetary authorities have been operating for an infinite number of periods and will honor commitment made in the past when choosing their optimal policies. This form of policy commitment is similar to the notion of optimality from a *timeless perspective* in the sense of Woodford [2003]

We define the Ramsey policy as the monetary policies under commitment which maximize the joint sum of intertemporal households' welfare for country H and country F. Formally, the Ramsey equilibrium is a set of 46 processes defined in the competitive equilibrium for $t \geq 0$ that maximize

$$\mathcal{W}_{World,0} = \mathcal{W}_{H,0} + \mathcal{W}_{F,0}$$

subject to the competitive equilibrium conditions (1)-(43) and the analogous of equations (1), (2), (5) for country F, $\forall t \succ -\infty$, given exogenous stochastic processes and the initial values of the variables listed above dated $t \prec 0$, as well as the values of the Lagrange multipliers associated with the constraints listed above dated $t \prec 0$.

The Ramsey formulation of optimal monetary policy cooperation is therefore computed by formulating an infinite-horizon Lagrangian problem of maximizing the conditional expected social welfare subject to the full set of non-linear constraints forming the competitive equilibrium of the model. The first order conditions to this problem could easily be obtained using symbolic Matlab procedures. This approach would in principle be useful to derive the optimal policy in the general case, with inefficient steady state and the full set of frictions described above. The Ramsey approach to optimal monetary policy in a two-country model with PCP and perfect risk sharing has been studied by Faia and Monacelli [2004].

However, the potential drawback of this tractable approach to compute optimal policy in general modeling frameworks, is the lack of transparency on the policy trade-offs embodied in the model and on the optimal resolution of them. In this paper, we precisely intend to give more insight on the implications of international friction for optimal monetary policy cooperation: we study some particular cases for which a first order approximation of the Ramsey solution can be written in terms of targeting rules like the ones Giannoni and Woodford [2003a,b] obtained in a closed-economy set-up.

Up to a first-order numerical approximation, the same targeting rules could be derived through the linear-quadratic approximation of the Ramsey problem as in Benigno and Woodford [2006]. While this method presents less of an interest in large DSGE where the intuition about the optimal design of monetary policy would anyway be difficult to gain from the linear-quadratic formulation of the Ramsey problem, it may prove more useful in our simple cases in order to make explicit the policy trade-offs from the second-order approximation of the welfare function.

In the rest of the paper, we will also expose the linear-quadratic approximation of the Ramsey problem to illustrate the properties of optimal monetary policy cooperation.

3 Simple cases

Our approach is mainly illustrative: the influence of price setting and incomplete financial markets on optimal monetary policy is studied through highly stylized model configurations in which analytical solutions can be derived. Our main contribution here is to examine in a unified framework the optimal monetary cooperation, under PCP versus LCP, with complete or incomplete financial market, under efficient versus inefficient shocks. Some papers are closely related to our analysis. Clarida et al. [2002] studied the welfare gains from international cooperation under PCP within a slightly different model. Results exposed in this section under PCP are very close to the analysis of Benigno and Benigno [2006] who studied a two-country New Keynesian model under perfect risk sharing, PCP and no home bias. With imperfect exchange rate pass-through, Smets and Wouters [2002] give some results about optimal monetary policy in a small open economy. Within NOEM studies, we extend and find similar results as Devereux and Engel [2006] who used a static framework where price are set one period in advance. Benigno [2001] examined optimal monetary policy in model with incomplete financial markets and PCP but did not cover the financial autarky case and the analytical solution for optimal policy which is presented here. Overall, we propose a unified treatment of a large range of issues. In particular, it is shown through the exposition of optimal targeting rules that international price setting matters concerning the choice of the price deflator for the inflation objective. Moreover, results on optimal exchange rate regime found in the NOEM literature are generalized in some directions.

In the following, we restrict the analysis to the case of efficient steady state which allows us to easily approximate the Ramsey problem by a linear-quadratic one, relying only on the first order expansion of the structural constraints to derive the second-order approximation of the aggregate welfare.

The fully symmetric determinist steady state, around which we will log-linearize the model, is associated with the case where all shocks are held at their unconditional mean, subsidies offsetting the monopolistic distortions in goods and labor markets, and net foreign assets are zero. Inflation rates are null in the Ramsey steady state. All price levels are equalized. In that context, PPP does hold and all macroeconomic aggregates are the same across countries. In what follows, lower case letters stand for the logarithmic deviation from steady state.

3.1 The flexible price equilibrium

The economies are affected by "efficient" shocks like technological shocks and inefficient shocks that lead in particular to a short run inflation/output gap tradeoff for the conduct of monetary policy. In this stylized model, those shocks are only rationalized through markup fluctuations in the goods market (following time-varying tax rates).

In the flexible price allocation, only efficient shocks are introduced. Moreover, financial markets are assumed to be complete in the flexible price equilibrium. As we will see later, such a definition of the flexible price equilibrium is consistent with the welfare-relevant gaps (i.e. log-deviation of an actual variable from its flexible price counterpart) for output, consumption and terms-of-trade.

In the absence of price stickiness, the allocation is independent of monetary policy and all firms set prices equal to a constant markup over marginal cost while real wages equal marginal rates of substitution between hours and consumption. Moreover, as the demand elasticity of the differentiated intermediate goods is the same for local sales and exports, firms have no incentive to discriminate and the law of one price holds. The flexible allocation is therefore strictly independent from the price setting rules. Consequently, internal terms of trade are equalized across countries and relative export margins remain constant.

As we will see, the sticky price supply curves depend on the flexible price equilibrium. So it is convenient to indicate with a "$-$" over a variable a flexible price allocation. Moreover, since the model is easily solved in terms of aggregate and relative variable, we define for any variable X, $X^W = \frac{X+X^*}{2}$ and $X^R = \frac{X-X^*}{2}$. Finally, we denote the technological shocks $a_t = \log\left(\varepsilon^A\right)$ and $a_t^* = \log\left(\varepsilon^{A*}\right)$.

It can easily be shown that the flexible price allocation is given by

$$\begin{array}{llll} \textit{output} & \overline{z}_t^W = \frac{1+\sigma_L}{\sigma_C+\sigma_L} a_t^W & \overline{z}_t^R = \frac{(1+\sigma_L)\vartheta}{1/2+\vartheta\sigma_L} a_t^R \\ \textit{consumption} & \overline{c}_t^W = \frac{1+\sigma_L}{\sigma_C+\sigma_L} a_t^W & \overline{c}_t^R = \frac{(n-1/2)(1+\sigma_L)}{\sigma_C(1/2+\vartheta\sigma_L)} a_t^R \\ \textit{terms of trade} & \overline{t}_t = \frac{1+\sigma_L}{1/2+\vartheta\sigma_L} a_t^R & \end{array}$$

with $\vartheta = 2n(1-n)\xi + \frac{(2n-1)^2}{2\sigma_C}$.

In the following a hat over a variable indicates the absolute deviation from its flexible price value. For example, $\widehat{z}^W = z^W - \overline{z}^W$ is the world output gap.

3.2 Redundant financial markets

Thereafter, we denote the efficient optimal risk-sharing gap, i.e. the gap that should be closed under optimal risk sharing, by $2\sigma_C \, \widetilde{c}_t^R - \widetilde{rer}_t$, where $\sigma_C \, \widetilde{c}_t^R = \sigma_C \, \widehat{c}_t^R - (n - 1/2) \, \widehat{t}_t^W$ and $\widetilde{rer}_t = \widehat{rer}_t - (2n-1) \, \hat{t}_t^W$.

Making use of the Euler equations for both countries and the modified uncovered interest rate parity equations leads to the following imperfect risk sharing condition, up to a first order approximation:

$$\mathbb{E}_t \Delta \left(2\sigma_C \widetilde{c}_{t+1}^R - \widetilde{rer}_{t+1} \right) = -\chi b_t$$

where $\chi = -\Psi'_X(0) \, \overline{\overline{C}}/2$ and b_t is the percentage deviation from steady state of the net foreign assets of country H.

The first-order approximation of the net foreign assets dynamics can then be written as

$$\begin{aligned}\beta b_t = {} & b_{t-1} - 2\frac{1-n}{\sigma_C} \left(\sigma_C \, \widetilde{c}_t^R - \widetilde{rer}_t/2 \right) + (1-n)(1 - 1/\sigma_C) \, \widetilde{rer}_t \\ & + 2\mathcal{K}\widehat{t}_t^W + 2\mathcal{K}\bar{t}_t\end{aligned}$$

where $\mathcal{K} = n(1-n)(\xi - 1) + (1-n)(n - 1/2)(1 - 1/\sigma_C)$.

Using the previous two equations, we see that financial markets are redundant in the model when $\xi = 1$ and $\sigma_C = 1$, or under PCP (implying that $\widetilde{rer}_t = 0$), when $\xi = 1$ and $n = 1/2$. In both cases, with zero initial net foreign assets, the economy behaves as if financial markets were complete and $2\sigma_C \, \widetilde{c}_t^R - \widetilde{rer}_t = 0$ at all times. Therefore, those assumptions will make irrelevant imperfections in the international financial markets for the international monetary policy cooperation. This point is well known in the NOEM literature (see for example, Corsetti and Pesenti [2001]).

3.3 Quadratic approximation of the aggregate welfare

We take a second order approximation of the aggregate welfare function $\mathcal{W}_{World,t}$ around a steady state in which taxation subsidies completely offset the monopolistic distortions in both countries and both markets. In this context, the flexible price allocation is the first best solution and there are no first order terms in the second-order expansion of the welfare function.

By neglecting terms independent from monetary policy, we can show that[1]

[1] The fully-fledge derivation is similar to the one exposed by Darracq Pariès [2003] within a closely related framework.

$$\mathcal{W}_{World,t} = -1/2\overline{\overline{\Lambda C}}\mathbb{E}_t\sum_{j=0}^{\infty}\beta^j\omega_{t+j}$$

where

$$\omega_t = (\sigma_C + \sigma_L)\left(\widehat{z}_t^W\right)^2 + n(1-n)\xi\left(\frac{\widehat{t}_t^2 + \widehat{t}_t^{*2}}{2}\right) + \sigma_C\left(\widehat{c}_t^R\right)^2 + \sigma_L\left(\widehat{z}_t^R\right)^2$$
$$+\frac{1}{2}\frac{\mu}{\mu-1}\left(\begin{array}{c}(\eta(1-n)+n)\frac{\pi_{H,t}^2}{\lambda_H} + (1-\eta)(1-n)\frac{\tilde{\pi}_{H,t}^2}{\lambda_F^*} \\ +(\eta^*(1-n)+n)\frac{\left(\pi_{F,t}^*\right)^2}{\lambda_F^*} + (1-\eta^*)(1-n)\frac{\tilde{\pi}_{F,t}^2}{\lambda_H}\end{array}\right)$$

where $\lambda_H = \frac{(1-\alpha_H)(1-\beta\alpha_H)}{\alpha_H}$ and $\lambda_F^* = \frac{\left(1-\alpha_F^*\right)\left(1-\beta\alpha_F^*\right)}{\alpha_F^*}$.

We therefore approximate the Ramsey problem defined in previous section by the linear-quadratic one which maximizes this intertemporal welfare function under the structural equations of the model. Note that, up to a first order approximation, the solution of the linear-quadratic problem is the same as the one of the Ramsey problem presented in section 2.9.

Even in this simple framework, monetary authorities face numerous tradeoffs. First, inefficient shocks induce mechanically an inflation/output gap tradeoff. Second, under LCP incomplete pass-through, it is impossible to both stabilize export margins and internal terms of trade misalignments. Finally, financial imperfections introduce an additional wedge in the risk sharing conditions so that real exchange rate and relative consumption stabilization become conflicting objectives.

Therefore, in general, under imperfect pass-through and incomplete financial markets, the optimal cooperative policy cannot achieve the first best allocation.The optimal plan always requires adjusting gradually the price levels and the nominal exchange rate.

We now turn to special cases highlighting the impact of different frictions on optimal monetary cooperation. The point is not to present a comprehensive analysis of the optimal policies but instead to develop illustrative configurations conveying the qualitative features.

3.4 The producer-currency-pricing case

Under producer-currency-pricing in both countries ($\eta = 1$ and $\eta^* = 1$, implying $\widetilde{rer}_t = 0$), elementary algebra shows that the dynamics of the world economy can be summarized by the following relationships:

$$2\sigma_C \mathbb{E}_t \left(\Delta \widetilde{c}_{t+1}^R\right) = -\chi b_t \tag{RS}$$

$$\beta b_t = b_{t-1} - 2(1-n)\widetilde{c}_t^R + 2\mathcal{K}\widehat{t}_t + 2\mathcal{K}\bar{t}_t \tag{BOP}$$

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + \lambda_H \left[(\sigma_C + \sigma_L)\widehat{z}_t^W + (1/2 + \sigma_L \vartheta)\widehat{t}_t + \mathcal{X}\sigma_C \widetilde{c}_t^R\right] + u_{H,t} \tag{ASH}$$

$$\pi_{F,t}^* = \beta \mathbb{E}_t \pi_{F,t+1}^* + \lambda_F^* \left[(\sigma_C + \sigma_L)\widehat{z}_t^W - (1/2 + \sigma_L \vartheta)\widehat{t}_t - \mathcal{X}\sigma_C \widetilde{c}_t^R\right] + u_{F,t}^* \tag{ASF}$$

$$\widehat{t}_t = \widehat{t}_{t-1} + \Delta s_t + \pi_{F,t}^* - \pi_{H,t} - \Delta \bar{t}_t \tag{TT}$$

$$\sigma_C\, \widetilde{c}_t^R = \sigma_C\, \widehat{c}_t^R - (n - 1/2)\,\widehat{t}_t$$

with $\mathcal{X} = 1 + (2n-1)\,\sigma_L/\sigma_C$. The markup shocks are given by $u_{H,t} = \lambda_H \log\left(\varepsilon_t^P\right)$ and $u_{F,t}^* = \lambda_F^* \log\left(\varepsilon_t^{P*}\right)$.

In this reduced form, all state variables are written in deviation from its flexible-price path and efficient shocks are introduced in the model through the flexible-price path of terms of trade.

The terms of trade misalignments (i.e. in deviation from the flexible price path) play a key role in this model driving a wedge between the inflation rates in both countries. They enter the aggregate supply equations through two different channels. First, workers negotiate on real wage measured with the consumer price index whereas producer price inflation rate depends on real wage measured with the producer price index. So when the price of foreign goods increases, workers wants higher salaries to compensate from lower real income, which pushes up local producer prices. Second, the expenditure-switching effect reflects the fact that an increase in the price of goods produced in one country relative to goods produced in the other boosts the demand for goods produced in the latter and hours worked by residents. They claim for higher wage so that producer inflation increases in this country. Notice that under complete financial markets (i.e. $\widetilde{c}_t^R = 0$), the introduction of home bias in the model does not change fundamentally the structure of the reduced form under PCP. It magnifies the expenditure-switching effect through the impact of terms of trade on relative consumption, as we see in the elasticity $\vartheta = 2n(1-n)\,\xi + \frac{(2n-1)^2}{2\sigma_C}$.

Finally, under incomplete financial markets, net foreign assets imbalance introduces a risk premium in the uncovered interest rate parity and drives a wedge between relative consumption and real exchange rate. In addition, the optimal risk-sharing gap enters real marginal costs in country H and F with opposite signs.

The welfare function also simplifies to

$$\omega_t = (\sigma_C + \sigma_L) \left(\widehat{z}_t^W\right)^2 + n\,(1-n)\,\xi \widehat{t}_t^2 + \sigma_C \left(\widehat{c}_t^R\right)^2 + \sigma_L \left(\widehat{z}_t^R\right)^2 + \frac{1}{2}\frac{\mu}{\mu-1}\left(\frac{\pi_{H,t}^2}{\lambda_H} + \frac{\left(\pi_{F,t}^*\right)^2}{\lambda_F^*}\right)$$

With complete financial markets, it even gives

$$\omega_t = (\sigma_C + \sigma_L) \left(\widehat{z}_t^W\right)^2 + \vartheta\,(1/2 + \sigma_L \vartheta)\,\widehat{t}_t^2 + \frac{1}{2}\frac{\mu}{\mu-1}\left(\frac{\pi_{H,t}^2}{\lambda_H} + \frac{\left(\pi_{F,t}^*\right)^2}{\lambda_F^*}\right)$$

This expression of the aggregate welfare function can also be related to what Benigno and Benigno [2003] obtain under some similar model assumptions. One minor contribution here is to allow for some degree of home bias in national consumption. Under PCP, this assumption increases the distortions associated with terms of trade misalignments: terms of trade affects relative output both through the traditional expenditure switching effect and through its impact on real exchange rate. Under complete financial markets, the social cost of those deviations are analogous to what we would find in model without preference bias but where the intratemporal elasticity of substitution is given by $\vartheta = 2n\,(1-n)\,\xi + \frac{(2n-1)^2}{2\sigma_C}$.

In that sense, the fundamental channels through which terms of trade affect the welfare function are not qualitatively modified by the home bias hypothesis: terms of trade misalignments are costly due to its impact on relative output gap and on relative labor supply.

The following result presents the targeting rules for optimal monetary policy cooperation.

Result 1 *Under PCP, the optimal policies are determined by the following equations*

$$\frac{\mu}{\mu-1}\pi_{H,t} = -\Delta \widehat{z}_t \text{ and } \frac{\mu}{\mu-1}\pi_{F,t}^* = -\Delta \widehat{z}_t^*$$

if and only if

(i) *financial markets are complete, or*

(ii) *incomplete financial markets and $\xi = 1$ and $\sigma_C = 1$, or*

(iii) *incomplete financial markets and $\xi = 1$ and $n = 1/2$.*

The design of the targeting rules is essentially valid under complete markets or when financial markets are redundant. In that case, not only do the monetary authorities choose the same optimal strategy as under complete markets, but they also achieve the same allocation. Benigno and Benigno [2006] provide a generalization of those targeting rules under perfect risk sharing and no home bias to the case of inefficient steady state.

Corollary 1 *Under the conditions stated in the previous result and following efficient shocks, the optimal cooperative policy achieves the flexible price allocation: there is no volatility of inflation and output gaps are closed. In that case, pure inflation targeting policies implement the optimal solution.*

In presence of cost-push shocks, the monetary authorities face a tradeoff between stabilizing the output gap or the inflation rate which prevents the optimal policy to reach the first best allocation.

Let us consider the behavior of the exchange rate under the optimal monetary policy cooperation. Following efficient shocks, the optimal policy replicates the flexible price allocation. Therefore, as the inflation rates are equal to zero and the "terms of trade gap" is closed, equation TT shows that the nominal exchange rate has to adjust to the required terms of trade path under flexible prices. However, in presence of inefficient shocks, the associated inflation/output gap tradeoff does not allow to fully stabilize the economies. The optimal monetary cooperation targets the producer price levels and the nominal exchange rate. A fixed exchange rate regime might even be optimal.

Result 2 *Under the assumptions of Result 1 and following efficient shocks, it is optimal to let the exchange rate freely adjust to the efficient fluctuations of international relative prices. Otherwise, an exchange rate management is needed. In particular, following inefficient shocks, it is optimal to fix it when* $\frac{\mu}{\mu-1} = 2\vartheta$.

Proof: combining the optimal policies and equation TT, and making use of $\hat{y}_t^R = \vartheta\hat{t}_t$, it is easy to show that the inflation rate differential realizes exactly the required terms of trade adjustment if $\frac{\mu}{\mu-1} = 2\vartheta$, leaving no role for exchange rate variations.

Overall, beyond the introduction of home bias, the results exposed here are similar to what is exposed in Benigno [2001] with incomplete risk sharing or Benigno and Benigno [2003] with perfect risk sharing. We are now going to illustrate more precisely the role of imperfect risk sharing on optimal monetary policy coordination in the extreme case of financial autarky.

3.5 Producer-currency-pricing and financial autarky

When the elasticity of the exchange risk premium with respect to net foreign assets χ goes to infinity, it becomes too expensive for agents to buy foreign bonds and, as a limit case, net foreign assets in both countries are permanently null. In this context, we can express the equation BOP in the following way:

$$c_t^R = (n\xi - 1/2)\, t_t$$

Compared with the perfect risk sharing relation $\sigma_C c_t^R = (n - 1/2)\, t_t$, the intratemporal elasticity of substitution ξ affects the correlation between relative consumption and the terms-of-trade. This correlation can be either positive or negative depending on $\xi > \xi_1 = \frac{1}{2n}$ or $\xi < \xi_1$.

Similarly, the relation between relative output and the terms-of-trade is given by

$$z_t^R = (n\xi - (n - 1/2))\, t_t$$

Here again, in comparison with the perfect risk sharing equivalent $z_t^R = \vartheta t_t$ where $\vartheta > 0$, the correlation between relative output and the terms-of-trade is positive when $\xi > \xi_2 = \frac{2n-1}{2n}$ and negative otherwise.

Corsetti et al. [2005] illustrate the role of the price elasticity of tradable goods under incomplete markets on the sign of the international transmission and of the correlation between relative consumption and real exchange rates. Within a simple endowment economy, they highlight the role of the cutoff point ξ_2 for the price elasticity, on the equilibrium volatility of the terms-of-trade. When ξ gets closer to ξ_2, the volatility of the terms-of-trade increases in response to relative output shocks, implying that there will be two values of ξ with opposite international transmission sign, which could yield the same volatility level. Moreover, since $\xi_1 > \xi_2$, the value of ξ associated with positive international transmission could also imply negative correlation between relative consumption and the real exchange rate. In our framework, with elastic supply curves and nominal rigidities, such cutoff point will obviously not be the same.

In the following, we are going to investigate the design of optimal monetary policy cooperation under PCP and financial autarky. This special case provides some perspective on the role of the intratemporal elasticity of substitution with incomplete financial markets.

Given financial autarky, we can re-write the marginal costs so that the reduced form of the model becomes

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + \lambda_H \left[(\sigma_C + \sigma_L) \widehat{z}_t^W + \mathcal{A}\widehat{t}_t \right] + \lambda_H \mathcal{X} \sigma_C \mathcal{K} \bar{t}_t + u_{H,t} \quad \text{(ASHA)}$$

$$\pi_{F,t}^* = \beta \mathbb{E}_t \pi_{F,t+1}^* + \lambda_F^* \left[(\sigma_C + \sigma_L) \widehat{z}_t^W - \mathcal{A}\widehat{t}_t \right] - \lambda_F^* \mathcal{X} \sigma_C \mathcal{K} \bar{t}_t + u_{F,t}^* \quad \text{(ASFA)}$$

$$\widehat{t}_t = \widehat{t}_{t-1} + \Delta s_t + \pi_{F,t}^* - \pi_{H,t} - \Delta \bar{t}_t \quad \text{(TT)}$$

where $\mathcal{A} = (1-n) + \sigma_C (n\xi - 1/2) + \sigma_L (n\xi - (n - 1/2))$.

As in the perfect risk sharing case, the terms-of-trade gap drives a wedge between the inflation rates of both countries (see equations ASHA and ASFA). However, with financial autarky, the elasticity of the real marginal cost with respect to the terms-of-trade $\mathcal{A}$ can be negative for $\xi < \xi_3 = \xi_2 + \frac{(1-n)(\sigma_C - 1)}{n\sigma_C + n\sigma_L}$. ξ_3 would represent the cutoff point of Corsetti et al. [2005] in our framework with financial autarky and flexible prices. With nominal rigidity, such value for ξ depends also on monetary policy.

The extreme financial market imperfections we consider in this section provides an illustration of the associated policy trade-offs: even following efficient shocks, the flexible-price terms-of-trade acts as cost-push shocks, pushing inflation rates in opposite directions. This introduces an additional trade-off between the stabilization of inflation rates and output gaps in particular.

Under financial autarky, the welfare function can be written as follows

$$\omega_t = (\sigma_C + \sigma_L) \left(\widehat{z}_t^W \right)^2 + \mathcal{B} \widehat{t}_t^2 + 2\mathcal{C} \widehat{t}_t \bar{t}_t + \frac{1}{2} \frac{\mu}{\mu - 1} \left(\frac{\pi_{H,t}^2}{\lambda_H} + \frac{\left(\pi_{F,t}^* \right)^2}{\lambda_F^*} \right)$$

where

$$\mathcal{B} = n(1-n)\xi + \sigma_C (n\xi - 1/2)^2 + \sigma_L (n\xi - (n - 1/2))^2$$

$$\mathcal{C} = \left[(n\xi - 1/2) + (n\xi - (n - 1/2))(2n - 1) \right] \frac{\mathcal{K}}{1-n}$$

Note that $\mathcal{B} > 0$ but $\mathcal{C}$ can potentially be either sign.

Result 3 *Under PCP and when $\chi \to \infty$, the optimal policies are determined by the following equations*

$$\frac{\mu}{\mu - 1} \frac{\pi_{H,t} + \pi_{F,t}^*}{2} = -\Delta \widehat{z}_t^W$$

$$\frac{\mu}{\mu - 1} \mathcal{A} \frac{\pi_{H,t} - \pi_{F,t}^*}{2} = -\mathcal{B} \Delta \widehat{t}_t - \mathcal{C} \Delta \bar{t}_t$$

The targeting rules for optimal monetary policy cooperation are the same as the ones of Result 1 when $\frac{\mathcal{B}}{\mathcal{A}} = \vartheta$ and $\mathcal{C} = 0$. This is the case when $\xi = 1$ and $\sigma_C = 1$ or when $\xi = 1$ and $n = 1/2$. The second targeting rule of Result 3 shows that, depending on the sign of $\mathcal{A}$, the inflation differential should be adjusted positively or negatively to the changes in the terms-of-trade gap.

Concerning the exchange rate adjustment, we can show numerically that there is a cutoff point for ξ around which the response of nominal exchange rate changes sign. This point does not coincide with ξ_3 in general. Besides, as in Result 2, we can easily show that following markup shocks, the nominal exchange rate remains constant when $\frac{\mu}{\mu-1} = 2\frac{\mathcal{B}}{\mathcal{A}}$.

Up to our knowledge, such an exposition of the financial autarky case is new in the literature on optimal policy in DSGE models and presents enlightening illustration of the role played by incomplete markets on optimal monetary policy cooperation. Benigno [2001] in particular did not derive any analytical solution in its treatment of optimal policy under imperfect financial markets.

We now turn to the local-currency-pricing case.

3.6 The local-currency-pricing case

In this section, results obtained by Devereux and Engel [2003, 2006] are partly revisited and extended, notably to the case of cost-push shocks. Our main contribution here is to derive explicit optimal targeting rules under the LCP assumption and examine the optimality of exchange rate regimes according to the typology of shocks.

The LCP hypothesis introduces two additional distortions in the model. First of all, the nominal exchange rate doesn't affect the internal terms of trade directly. Thus, the expenditure-switching role of exchange rate is dampened by the stickiness of import prices: internal terms of trade are almost immune from monetary policy. However, exchange rate impacts instantaneously the relative export margins of producers, which induces some second round effects on inflation rates. This transmission mechanism conveys a second source of distortion. The variability of relative export margins implies some undesirable fluctuations of the real exchange rate. Of course, under PCP, the real exchange rate moves in line with the terms of trade when there is a home bias in the national consumption. But under LCP, there is an additional source of deviation from PPP. This further deteriorates the international consumption risk sharing. Unlike the PCP case, it turns out that the LCP model is significantly modified by the home bias assumption. In particular, it changes the qualitative impact of internal terms of trade and relative export margins on real marginal costs. Consequently, it may be helpful to restrain the analysis to a no home bias case.

Under this restriction, the only state variables of the model are the world consumption gap (note that $\hat{c}_t^W = \hat{y}_t^W$), the real exchange rate and the CPI inflation rates. The dynamics of the world economy can be described by the following relations.

$$\mathbb{E}_t \Delta \left(2\sigma_C \widehat{c}_{t+1}^R - rer_{t+1} \right) = -\chi b_t \tag{LRS}$$

$$\beta b_t = b_{t-1} - \frac{1}{\sigma_C} \left(\sigma_C \, \widehat{c}_t^R - rer_t/2 \right) + 1/2 \left(1 - 1/\sigma_C \right) rer_t + 2\mathcal{K}\widehat{t}_t + 2\mathcal{K}\bar{t}_t \tag{LBOP}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda_H \left[\left(\sigma_C + \sigma_L \right) \widehat{c}_t^W + rer_t/2 \right] + u_t \tag{CASH}$$

$$\pi_t^* = \beta \mathbb{E}_t \pi_{t+1}^* + \lambda_F^* \left[\left(\sigma_C + \sigma_L \right) \widehat{c}_t^W - rer_t/2 \right] + u_t^* \tag{CASF}$$

$$rer_t = rer_{t-1} + \Delta s_t + \pi_t^* - \pi_t \tag{RER}$$

where the markup shocks are given by

$$u_t = \lambda_H \left(\log \left(\varepsilon_t^P \right) + \log \left(\varepsilon_t^{P*} \right) \right) \text{ and } u_t^* = \lambda_F^* \left(\log \left(\varepsilon_t^P \right) + \log \left(\varepsilon_t^{P*} \right) \right).$$

Given the aggregate output gap, internal terms of trade misalignments have no impact on consumer price indexes, but they still push away import prices from producer prices. Under LCP, there is no direct pass-through of nominal exchange rate on internal terms of trade. The immediate transmission mechanism of exchange rate relies on its impact on relative export margins and on the real exchange rate. Precisely, it is now the real exchange rate that pushes the inflation rates in opposite directions through the modified aggregate supply curves. This canonical representation of the economy under LCP will be useful in drawing the intuition about the properties of the optimal policy. Since the real exchange rate determines relative consumption, we can already notice that, compared to the PCP reduced-form model, CPI inflation rates and consumption gaps replace PPI inflation rates and output gaps as the fundamental state variables driving the economy.

Result 4 *The internal terms of trade and the net foreign assets are independent from monetary policy if and only if*

(i) *financial markets are complete and $n = 1/2$, or*

(ii) *financial markets are incomplete, $n = 1/2$ and $\sigma_C = 1$.*

Proof: Subtract equation the producer and import price setting equations in both countries, and replace the inflation rate differentials by the internal terms of trade dynamic equations. Furthermore, injecting LRS in LBOP to substitute for the term $\tilde{c}_t^R - rer_t/2$, one easily obtains three relations linking the internal terms of trade gaps and the net foreign assets to exogenous shocks.

In that case, it becomes clear that monetary policy cannot stabilize both producer prices and import prices (due to imperfect pass-through), and both relative consumption and real exchange rate (due to incomplete markets). This property was also singled out by Benigno [2004].

Since monetary policy has no control on internal terms of trade and therefore on inflation differentials $\pi_{F,t} - \pi_{H,t}$ and $\pi_{F,t}^* - \pi_{H,t}^*$, the loss function boils down to

$$\omega_t = (\sigma_C + \sigma_L) \left(\widehat{c}_t^W\right)^2 + \sigma_C \left(\widehat{c}_t^R\right)^2 + \frac{1}{2\lambda_H}\frac{\mu}{\mu-1}\pi_t^2 + \frac{1}{2\lambda_F^*}\frac{\mu}{\mu-1}\left(\pi_t^*\right)^2$$

As the real exchange rate is directly connected to the consumption in the case of complete financial markets, the reduced form of the structural equations and the previous loss function seem to indicate that monetary authorities cannot do better than stabilizing the CPI inflation rates and the consumption gaps. This intuition is confirmed by the following result.

Result 5 *The optimal cooperative policies is given by*

$$\frac{\mu}{\mu-1}\pi_t = -\Delta\widehat{c}_t \text{ and } \frac{\mu}{\mu-1}\pi_t^* = -\Delta\widehat{c}_t^*$$

if and only if

(i) *financial markets are complete and $n = 1/2$, or*

(ii) *financial markets are incomplete, $n = 1/2$ and $\sigma_C = 1$.*

Whereas, under producer-currency-pricing, the monetary authorities adjust the producer price inflation rate in response to the output gap fluctuations, when prices are set in the consumer currency, it is optimal to adjust the consumer price level to the variation of the "consumption gap". We have already seen that monetary policy has no impact on inflation differential between import price and producer price and cannot alleviate the distortions associated with terms of trade misalignments. Consequently, monetary stabilization works only on global consumption gap (equal to the aggregate output gap), the real exchange rate and the CPI inflation rates.

Note that under incomplete financial markets, the conditions $n = 1/2$ and $\sigma_C = 1$ do not imply that monetary authorities achieve the same allocation than under complete markets. This would only be the case for $\xi = 1$ which leads to redundant asset markets.

The derivation of the targeting rules under LCP is a novelty of our paper. Benigno [2004] touched upon many issues related to the effect of imperfect pass-through on the macroeconomic transmission of shocks and on exchange rate persistence but he did not investigate the welfare approximation nor the optimal policy.

Corollary 2 *Following efficient shocks, the optimal solution consists in completely stabilizing the consumer price levels and closing the consumption gaps if and only if*

(i) *financial markets are complete and* $n = 1/2$*, or*

(ii) *financial markets are incomplete,* $n = 1/2$, $\sigma_C = 1$ *and* $\xi = 1$.

Pure CPI inflation targeting implements the optimal policy.

Proof: Replace the CPI inflation rates in CASH and CASF using the optimal policies and the real exchange rate using the optimal risk sharing condition (which also holds under imperfect risk sharing with $\sigma_C = 1$ and $\xi = 1$). This shows that the consumption gaps are systematically closed.

Following efficient shocks and perfect risk sharing, the optimal plan succeeds in eradicating the social costs of deviations from the natural level of aggregate output and failures of PPP. Consumption gaps are closed, consumer price levels remain constant and exchange rate is fixed. However, as we have already mentioned, there still exists a tradeoff between import price and producer price stabilization.

We consider now the optimal dynamics of the exchange rate. Under LCP, the law of one price does not hold and the expenditure-switching role of exchange rate is muted. Therefore, it may seem quite appropriate to limit exchange rate variations in order to minimize the welfare costs associated with these distortions, since those fluctuations may not provide some compensating gains in terms of stabilization. And this property is likely to prevail independently from the originating shocks. The following result shows that a fixed exchange rate regime is optimal under certain conditions.

Result 6 *Under the assumptions that of corollary 2, the optimal cooperative policy imposes a fixed exchange rate regime if and only if*

i) *shocks are efficient, or*

ii) *shocks are inefficient shocks and* $\frac{\mu}{\mu-1}\sigma_C = 1$.

Proof: Following efficient shocks and the assumptions of Result 6, the optimal policy fully stabilizes the consumer price levels and closes the consumption gaps. Furthermore, using equations CASH and CASF, we see that the real exchange rate remains constant. So equation RER implies that the nominal exchange rate is fixed. Otherwise, reminding that the purchasing power parity holds in the flexible equilibrium without preference bias, we make use of the optimal risk sharing condition (which holds under the assumptions of Result 6) to show that the optimal real exchange rate variations are matched by the inflation rate differential if $\frac{\mu}{\mu-1}\sigma_C = 1$.

Devereux and Engel [2003, 2006] provide similar conclusions as in Result 6 regarding the optimality of fixed exchange rate regime with efficient shocks and LCP. Here, we extended the analysis to a dynamic framework and more importantly to inefficient shocks, showing that in general, some degree of exchange rate flexibility is needed while a fixed exchange rate allocation could only arise with a specific combination of model parameters.

Note however that, in our framework, the optimality of fixed exchange rate under LCP is obtained under restrictive assumptions. In particular, introducing a preference bias, even with complete financial markets breaks the result: monetary authorities can have an impact on the internal terms of trade and therefore face a tradeoff between the stabilization of the terms of trade gaps which requires some exchange rate flexibility and the stabilization of export margins which moves inefficiently with exchange rate fluctuations. With high degree of home bias, optimal policy under LCP may even require more exchange rate adjustment than under PCP in order to promote efficient fluctuations of real exchange rate with its more limited leverage on the terms of trade.

4 Conclusion

This paper aimed at providing some stylized benchmarks for optimal monetary policy cooperation. We derived optimal targeting rules in some specific model configurations to illustrate the role of imperfect exchange rate pass-through, incomplete international financial markets and home bias in consumption. Such simple cases should also prove a useful guide for the analysis of optimal monetary policy cooperation in large DSGE models.

In particular, we show that the introduction of ”pricing-to-market” changes or extends previous results found in the literature. Under ”local-currency-pricing”, the monetary authorities should target the consumer price index. A pure CPI inflation targeting strategy implements the optimal outcome when shocks are efficient. An analogous result holds under ”producer-currency-pricing” concerning the optimality of PPI inflation targeting.

Moreover, a fixed exchange rate regime may be optimal under LCP in order to alleviate distortions associated with failures of the law of one price. Under PCP, a flexible exchange rate regime is optimal following efficient shocks. However, the presence of cost-push shocks reinforces the case for exchange rate management.

In order to explore the effect of imperfect risk sharing on optimal cooperation, the case of financial autarky under PCP shows that even with efficient shocks the first best allocation cannot be achieved. This special case also provides some perspective on the role of the intratemporal elasticity of substitution with incomplete financial markets in shaping the optimal response of international relative prices.

Our analysis reveals the lack of robustness of results about optimal monetary policy in open economies and the importance of correctly modeling international financial market structure and the international price setting.

References

S. Adjémian, M. Darracq Pariès, and S. Smets. A quantitative perspective on optimal monetary policy cooperation between the us and the euro area. mimeo, European Central Bank, 2007.

G. Benigno. Real exchange rate persistence and monetary policy rules. *Journal of Monetary Economics*, 51(3):473–502, 2004.

G. Benigno and P. Benigno. Price stability in open economies. *Review of Economic Studies*, 70: 743–764, 2003.

G. Benigno and P. Benigno. Designing targeting rules for international monetary policy cooperation. *Journal of Monetary Economics*, 53:473–506, 2006.

P. Benigno. Price stability with imperfect financial integration. Discussion Papers 2854, CEPR, 2001.

P. Benigno and M. Woodford. Linear-Quadratic Approximation of Optimal Policy Problems. Working Paper 12672, NBER, November 2006.

R. Clarida, J. Gali, and M. Gertler. A simple framework for international monetary policy analysis. *Journal of monetary economics*, 49(2):879–904, 2002.

G. Corsetti, L. Dedola, and S. Leduc. International risk sharing andthe transmission of productivity shocks. International Finance Discussion Papers 826, Board of Governors of the Federal Reserve System, February 2005.

G. Corsetti and P. Pesenti. Welfare and macroeconomic interdependance. *Quarterly Journal of Economics*, 116:421–445, 2001.

G. Corsetti and P. Pesenti. International dimensions of optimal monetary policy. Discussion Papers 3349, CEPR, 2002.

M. Darracq Pariès. *Price Setting and Optimal Monetary Policy: A New Keynesian Perspective*. in ”Exchange Rates and Business Cycles: A New Open Economy View”, J.O. Hairault and T.Sopraseuth, Eds. Routledge, 2003.

M. Devereux and C. Engel. Monetary Policy in the Open Economy Revisited: Exchange rate Flexibility and Price Setting Behavior. *Review of Economic Studies*, 60:765–783, 2003.

M. Devereux and C. Engel. Expenditure switching vs. real exchange rate stabilization: Competing objectives for exchange rate policy. Working Paper 614, ECB, 2006.

E. Faia and T. Monacelli. Ramsey monetary policy and international relative prices. Working Paper 344, European Central Bank, 2004.

M. Gali and T. Monacelli. Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72(3):707–734, 2005.

M. Giannoni and M. Woodford. Opitmal Interest-Rate Rules: I. General Theory. Working Paper 9419, NBER, January 2003a.

M. Giannoni and M. Woodford. Optimal Interest-Rate Rules: II. Applications. Working Paper 9420, NBER, January 2003b.

M. Obstfeld and K. Rogoff. Risk and exchange rates. Working Paper 6694, NBER, August 1998.

M. Obstfeld and K. Rogoff. Exchange rate dynamics redux. *Journal of Political Economy*, 103(3): 624–660, 2000.

M. Obstfeld and K. Rogoff. Global implications of self-oriented national monetary rules. *Quaterly Journal of Economics*, 117:503–536, 2002.

J. Rotemberg and M. Woodford. An optimization-based econometric model for the evaluation of monetary policy. NBER Macroeconomics Annual 12, 1997.

S. Schmitt-Grohe and S. Uribe. Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model. Working Paper 11854, NBER, December 2005.

F. Smets and R. Wouters. Openness, imperfect exchange rate pass-through and monetary policy. *Journal of Monetary Economics*, 49(5):947–981, 2002.

A. Sutherland. International monetary policy coordination and financial market integration. Working paper 174, ECB, 2002.

M. Woodford. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, 2003.

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