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INFLATION DYNAMICS AND REGIME SHIFTS

by Julia Lendvai

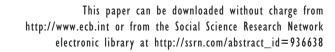




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In 2006 all ECB publications feature a motif taken from the €5 banknote.





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#### Abstract

This paper extends the New Keynesian model to allow for stochastic shifts in the monetary policy regime. Agents cannot observe the regime and use a Bayesian learning rule to make optimal inferences. Price setting is adapted to this environment: lagged expectations about monetary policy influence the current inflation rate through an indexation rule. No structural inflation persistence is assumed. We show that this model can capture stylized facts about short-run inflation dynamics both in periods of transition and in stable environments. The role of expectations increases after regime shifts. This creates a link between the degree of inflation persistence and the stability and transparency of monetary policy. Thereby, our model can explain observed changes in inflation persistence.

*Keywords:* Inflation dynamics, regime shifts, Bayesian learning, inflation persistence *JEL*: E30, E31, E32

## Non-technical Summary

Recent empirical studies on the conduct of monetary policy have pointed out significant shifts in the policy over the past decades. These shifts seem to have concerned the systematic policy, the policy objectives as well as the monetary policy shocks.

Parallel to these shifts in the conduct of monetary policy, the dynamics of inflation has changed over time as well: both the volatility and the persistence of inflation have decreased since the end of the 70s. A growing number of papers report evidence suggesting a link between changes in inflation dynamics and the stability and transparency of the monetary policy regime.

Notwithstanding these empirical findings, standard monetary macroeconomic models assume that the monetary policy regime is fixed. To reproduce observed facts about inflation dynamics such as inflation persistence and real costs of disinflations, many of these studies model inflation as being structurally persistent, i.e. as being sluggish due to the backwardlooking price setting of firms.

This paper takes a different approach by extending the standard New-Keynesian model to allow for shifts in the monetary policy regime and for incomplete information on the part of private agents. Firms' staggered price setting is adapted to a world in which the central bank's target can shift and is not fully credible. We do at the same time not rely on structural inflation persistence. This model is used to study the impact of changes in the conduct of monetary policy on inflation dynamics and on the dynamic link between inflation and output.

Specifically, a shift in the monetary policy regime is captured by changes in the central bank's long-run inflation target and in the volatility of the control error. Private agents have limited information about the type of the central bank and therefore face a signal extraction problem. They use a Bayesian learning rule to make optimal inference about the current regime. Since learning is gradual, this mechanism introduces an additional source of inflation persistence in our model.

As we discuss, this source of inflation persistence does not play an important role in stable periods. When agents had the time to learn about the type of the monetary policy and no major change in policy occurs, the propagation of shocks is similar to that in an economy where agents can directly observe the regime. In contrast, the role of learning increases in periods of great uncertainty which typically follow a shift in the monetary policy regime.

Our main findings are as follows.

First, we show that our model generates realistic degrees of inflation persistence in stable periods.

Second, in line with previous findings, the learning mechanism in our model contributes to reproducing persistent and costly disinflations, and helps to account for systematic inflation forecast errors during disinflation periods.

Third, on the top of previous findings, we show that our model can endogenously explain observed changes in inflation persistence. Since learning is a major source of inflation persistence in the model and since the role of learning increases in periods of uncertainty about the regime, inflation persistence is also predicted to be higher in these unstable periods than in more stable periods. Thereby, the signal extraction problem in our model creates a link between inflation persistence and the lack of transparency and stability of monetary policy.

As we discuss, this mechanism by which the conduct of monetary policy influences inflation dynamics is an alternative to the standard assumption of structural backward-looking price setting both in explaining real costs of disinflations and in generating inflation persistence. While the standard assumption of backward-looking price setting behavior is however, by itself, not sufficient to account for observed changes in inflation dynamics over time, our model tracks well the historic changes in inflation persistence between relatively stable and relatively unstable periods.

## 1 Introduction

In recent years, a growing number of empirical studies have pointed out shifts in the conduct of monetary policy. Clarida, Galì & Gertler (2000) document changes in the post-war U.S. systematic monetary policy reaction function. Sims & Zha (2006) estimate regime shifts for the U.S. monetary policy and find significant changes over the past decades which predominantly bear on the volatility of the control error. Ireland (2005) reports changes in the Fed's inflation target.<sup>1</sup>

In addition, there has been growing evidence suggesting that the conduct of monetary policy has an influence on the nature of inflation dynamics. According to these studies, the persistence and the volatility of inflation have changed over time; moreover, these changes appear to be linked to the stability and transparency of the monetary policy regime; see e.g. Cogley & Sargent (2001 and 2005), Goodfriend & King (2001), Levin & Piger (2004), Couvoisier & Mojon (2005), Benati (2005).

Notwithstanding these empirical findings, standard monetary macroeconomic models explicitly or implicitly assume that the monetary policy regime is fixed.<sup>2</sup> To reproduce observed facts about inflation dynamics such as inflation persistence and real costs of disinflations, many of these studies model inflation as being structurally persistent, i.e. as being sluggish due to firms' backward-looking price setting behavior.<sup>3</sup>

This paper takes a different approach by extending the standard New-Keynesian model<sup>4</sup> to allow for shifts in the monetary policy regime and for incomplete information on the part of private agents. Firms' price setting is adapted to a world in which the central bank's target can shift and is not fully credible. We do at the same time not rely on structural inflation persistence. This model is used to study the impact of changes in the conduct of monetary policy on inflation dynamics and on the dynamic link between inflation and output.

Specifically, we relax the assumption of a constant, perfectly observed and fully expected inflation target and model shifts in the monetary policy regime as changes in the central bank's long-run target and in the volatility of the control error. Private agents have limited

<sup>&</sup>lt;sup>1</sup>For other studies emphasizing changes in the inflation target and/or the volatility of monetary policy shocks over time see e.g. Cogley & Sbordone (2005), Kozicki & Tinsley (2005), Primiceri (2005), Justiniano & Primiceri (2006).

<sup>&</sup>lt;sup>2</sup>Some of the mentioned models allow for shifts in the inflation target. At the same time, the effects of these shifts are neutralized by price setting assumptions. See e.g. Smets & Wouters (2004).

<sup>&</sup>lt;sup>3</sup>Throughout the entire paper, we use 'structural' or, equivalently, 'intrinsic' inflation persistence to refer to the fact that inflation directly depends on its past value.

 $<sup>{}^{4}</sup>$ See e.g. Galí (2003) or Woodford (2003).

information about the type of the central bank: they cannot observe the current regime and perceive shifts in the regime as a stochastic process which we specify as a two-state Markov switching process. Agents face a signal extraction problem. They use a Bayesian learning rule to make optimal inference about the current regime; this inference is then used to form expectations about the future. Similar specifications are suggested by Andolfatto & Gomme (2003), Leeper & Zha (2003) and Schorfheide (2005).<sup>5</sup> To avoid drastic policy changes after a regime shift, we allow the central bank to smooth its actions by a simple convergence rule.

We build on Calvo-type staggered price setting on the part of firms. In the Phillips curve we suggest, lagged expectations about monetary policy influence the inflation rate because of the rule-of-thumb pricing behavior of a fraction of firms. This modified New Keynesian Phillips Curve (NKPC) combines features of the standard New Keynesian and of the New Classical theories.

In addition to the usual sources of inflation persistence, such as staggered price setting combined with systematic monetary policy, in our model, the uncertainty about the monetary policy regime and Bayesian learning also contribute to generating inflation persistence. Analyzing the dynamic predictions of this model in a disinflation experiment (regime shift) and in response to monetary policy shocks within a given regime, we find the following results.

First, we show that our model is able to reproduce persistent and costly disinflations, and to account for systematic deviations of the forecast from the realized inflation during disinflation periods. In addition, our model generates a trade-off between the speed of disinflation and the sacrifice ratio. These findings are broadly in line with a large strand of literature studying the implications of incomplete information and learning for disinflations.<sup>6</sup>

Second, for stable regimes, i.e. when agents have learned about the type of the central bank and no shift occurs, we confirm findings of Leeper & Zha (2003) and Schorfheide (2005) according to which learning does not play an important role in the propagation of typ-

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 $<sup>{}^{5}</sup>$ For other papers on monetary policy assuming learning without departing from rationality see e.g. Erceg & Levin (2003) and Collard & Dellas (2005). These authors use signal extraction based on Kalman filter. In addition, since Evans & Honkapohja (2001) an extensive learning literature has developed in which some forms of irrational expectations are assumed.

<sup>&</sup>lt;sup>6</sup>Our disinflation analysis is most closely related to works by Andolfatto & Gomme (2003) and Erceg & Levin (2003). Both model incomplete information and learning by a signal extraction problem. Andolfatto & Gomme use Markov-switching process and Bayesian learning but their price setting assumptions are different from ours. Erceg & Levin build on Taylor contracts and on Kalman filtering. In addition, there is a large disinflation literature departing from the assumption of rational expectations. See e.g. Ball (1995), Goodfriend & King (2005), Nicolae & Nolan (2006), Nunes (2005).

ical monetary policy interventions. However, in contrast to previous findings, we show that these results can drastically change in periods of uncertainty which typically follow a regime shift. When agents are confused about the regime, the role of learning increases. Sizeable expectations-formation effects then significantly change the impact of typical interventions during the transition process.

Third, we show that our model can endogenously explain observed changes in inflation persistence. Since learning is a major source of inflation persistence in the model and since the role of learning increases in periods of uncertainty about the regime, inflation persistence is also predicted to be higher in these unstable periods than in more stable periods. Thereby, the signal extraction problem in our model creates a link between inflation persistence and the lack of transparency and stability of monetary policy.

As we discuss, the mechanism by which the monetary regime's stability and transparency influences inflation dynamics is an alternative to the assumption of structural backwardlooking price setting both in explaining real costs of disinflations and in generating inflation persistence. While the assumption of backward-looking price setting is however, by itself, not sufficient to account for observed changes in inflation dynamics over time, our model tracks well the historic changes in inflation persistence between relatively stable and relatively unstable periods. Our findings are in line with those reported by Erceg & Levin (2003), Collard & Dellas (2005), Orphanides & Williams (2005) and Milani (2005) who similarly point out that models with incomplete information and private agents' learning about the monetary policy can generate inflation persistence and reproduce realistic inflation and output dynamics without or with relatively little intrinsic inflation persistence. However, none of these papers discusses variations in the degree of inflation persistence and the role of policy stability and transparency therein.<sup>7</sup>

The remainder of this paper is organized as follows. Section 2 briefly describes stylized facts about U.S. inflation dynamics and presents evidence for the varying degree of inflation persistence over the past decades. Section 3 outlines the model. Section 4 presents the implications of a regime shift from high to low inflation target. Section 5 presents the impact of intra-regime interventions in stable and transition periods. Section 6 discusses the effect of regime shifts and transparency on inflation persistence and Section 7 concludes.

 $<sup>^{7}</sup>$ Erceg & Levin (2003) focus on disinflation episodes, while the other cited papers are concentrating on stable monetary policy regimes only.

## 2 Stylized Facts

This section describes key properties of U.S. inflation dynamics over the past 35 years.

The upper panel in Figure 1 displays the evolution of the annualized quarterly inflation rate and the output gap between 1970Q1 and 2005Q1. The bottom panel shows the evolution of the inflation rate and the one quarter ahead inflation forecast.<sup>8</sup> Descriptive statistics of the inflation rate and the forecast errors are shown in table 1 for the entire sample and for three different subperiods.<sup>9</sup> The following aspects are worth noting.

First, average inflation was the highest in the seventies, it decreased at the beginning of the eighties and it has been even lower since the beginning of the nineties.

Second, disinflation periods seem to coincide with a persistent contraction in real production. It is broadly acknowledged that the disinflation process itself contributed to some extent to the decline in the output gap during these periods.

Finally, as shown by the difference between forecasted and realized inflation, the increasing inflation of the 70s was systematically under-predicted, while during the disinflation periods of the Volcker era but also at the beginning of the 90s, future inflation was systematically overestimated. Root mean squared inflation forecast errors for the different subperiods suggest that uncertainty about future inflation was the highest in the 70s and at the beginning of the 80s, it has decreased in the second half of the 80s and it has been even lower since the beginning of the 90s. Inflation uncertainty as captured by forecast errors is attributed to agents' uncertainty about the monetary policy objectives and / or the monetary policy's lack of credibility by e.g. Evans & Wachtel (1993), Dotsey & DeVaro (1995), Erceg & Levin (2003) and Goodfriend & King (2005).

In addition, parallel to changes in mean inflation and in the root mean squared forecast error, the persistence of inflation has also varied over the past decades. Table 2 displays our estimates for the degree of persistence of the CPI and GDP deflator inflation for the entire

<sup>&</sup>lt;sup>8</sup>Inflation rate in the upper panel is CPI inflation rate. CPI data from IMF International Financial Statistics. Output gap data from OECD. The bottom panel inflation is the annualized quarterly growth of the GDP deflator. The forecast data is taken from the Survey of Professional Forecasters, median growth rate of GDP price index (before 1996 GDP implicit deflator, before 1992 GNP implicit deflator). Source: Federal Reserve Bank Philadelphia. CPI forecast was not available before 1981. Livingston forecast errors for 6 months ahead CPI show similar pattern.

<sup>&</sup>lt;sup>9</sup>As pointed out in Levin & Piger (2004), there is broad agreement in the literature that there was a break in inflation dynamics around the mid-80s. They estimate structural breaks for US GDP deflator and CPI inflation series and find significant breaks in the intercept around 1991. Our subsamples follow their estimations.

period 1970Q1 to 2005Q1 and for the three subperiods. Persistence is measured as the sum of autoregressive coefficients.<sup>10</sup> Specifically, we estimate the following equation by OLS:

$$\pi_t = \mu + \rho \pi_{t-1} + \sum_{j=1}^p \phi_j \Delta \pi_{t-j} + \varepsilon_t,$$

where  $\rho$  can be shown to be the sum of the autoregressive coefficients of the original  $\pi_t$  series' AR(p) process. Lag selection is based on the Akaike information criterion allowing for a maximum lag length of 6 quarters. Median unbiased estimates and 90% confidence intervals of the coefficient  $\rho$  were computed using Hansen's (1999) grid bootstrap procedure.<sup>11</sup>

The following results stand out.

First, the degree of persistence estimated for the entire sample is significantly higher than the ones estimated over separate subsamples. This is to a large extent due to the fact that ignoring shifts in mean inflation biases upwards the autocorrelation coefficients' estimates. This result has also been pointed out by Levin & Piger (2004) and Courvoisier & Mojon (2005).

Second, we confirm findings of recent empirical literature according to which the degree of inflation persistence has changed substantially over the sample period.<sup>12</sup> In particular, inflation was relatively persistent in the 70s and early eighties. Persistence has however decreased in the second half of the eighties. Since the early 90s, inflation appears to be close to white noise.

Finally, it is important to note that the period of high inflation persistence corresponds to a period of relatively high uncertainty about the future inflation rate as captured by root mean squared forecast errors, while low inflation persistence coincides with periods of relative certainty about the future inflation. In as far as inflation expectations are influenced by monetary policy, this suggests a link between inflation persistence and the ability of monetary policy to anchor expectations.

<sup>&</sup>lt;sup>10</sup>The same measure of persistence is used e.g. in Clark (2006), Levin & Piger (2004) or Benati (2005). This measure was advocated by Andrews & Chen (1994), who show that the sum of autoregressive coefficients is directly related to two alternative measures: the cumulative impulse response function and the spectrum at zero frequency.

<sup>&</sup>lt;sup>11</sup>The sampling distribution of the t-statistic  $t = \frac{\hat{\rho} - \rho}{S(\hat{\rho})}$  was simulated over a grid of 101 possible true values of  $\rho$  over an interval given by the sample persistence estimate plus or minus four OLS standard errors. For each possible value in the grid, 1000 replications were executed. Estimations use grid-bootstrap RATS procedure written by Clark (2006) available at http://econ.queensu.ca/jae/2006-v21.5/clark

<sup>&</sup>lt;sup>12</sup>See e.g. Taylor (2000), Cogley & Sargent (2001 and 2005), Levin & Piger (2004).

A link between U.S. inflation persistence and the conduct of monetary policy has also been pointed out by Cogley & Sargent (2001 and 2005). Estimating a drifting coefficient VAR model for post-war data, Cogley & Sargent find a strong positive correlation between the inflation target and the persistence of inflation. The authors point out that changes in the conduct of monetary policy "may have contributed to the rise and fall of inflation as well as to changes in its persistence".

Similar findings are reported by Benati (2005) for the 20th century U.K. inflation rate. Benati emphasizes that inflation persistence varies across monetary policy regimes. In particular, when monetary policy provided strong nominal anchors, inflation seems not to have been persistent at all (before WWI and during the inflation targeting regime since 1992). The less strong the nominal anchor in place, the higher the degree of inflation persistence is found to be. Benati argues that his findings provide evidence in favor of the notion that the ability of monetary policy to provide a credible nominal anchor is a key determinant of the degree of inflation persistence.

In the remaining sections, we present a model which is consistent with these stylized facts. In particular, it can reproduce real output costs of disinflations and realistic degrees of inflation persistence. In addition to these features, it can also endogenously explain changes in inflation persistence. Learning about unobservable monetary policy regimes is a major source of inflation persistence in our model. The role of learning increases in periods of high uncertainty about the monetary regime. This creates a link between persistence and the lack of transparency and stability in monetary policy.

## 3 The Model

The model economy consists of a representative household, a continuum of monopolistically competitive firms and the monetary policy authority. The model presented in this paper departs from the standard New Keynesian monetary DSGE model<sup>13</sup> in assumptions about the formulation of monetary policy, about firms' price setting behavior and about agents' information structure. Our model is closely related to the one outlined in Schorfheide (2005) with the most important differences lying in the monetary policy rules and the price setting behavior of firms.

 $<sup>^{13}</sup>$ See e.g. Galí (2003) or Woodford (2003).

#### 3.1 Monetary policy

Most of the literature assumes that the monetary policy regime is fixed and that the central bank's long-run inflation target is constant, known and fully anticipated by private agents. We relax these assumptions and consider monetary policy regimes as being subject to changes over time. In addition, we assume that private agents perceive the evolution of the monetary regimes as a stochastic process.

Specifically, a monetary policy regime is defined by the vector consisting of the long-run inflation target,  $\pi^*(s_t)$ , and the volatility of the control error  $\sigma_{(s_t)}^2$ . Two regimes are considered. One is characterized by a high long-run inflation target  $\pi^{*H}$  and loose control of the instrument as captured by high volatility of the monetary policy shock  $\sigma_H^2$ ; this regime will be referred to as 'high-target' regime  $(s_t = H)$ . The other regime is, vice versa, characterized by the vector  $(\pi^{*L}; \sigma_L^2)$ , i.e. a low inflation target and strict control of the instrument ('low-target' regime,  $s_t = L$ ).

Following Leeper & Zha (2003), Andolfatto & Gomme (2003) and Schorfheide (2005), private' agents perception of the law of motion governing monetary policy regimes is modeled as a two-state Markov-switching process. The transition probabilities between the regimes are contained in the transition matrix  $\Phi = \begin{bmatrix} \phi_{hh} & \phi_{lh} \\ \phi_{hl} & \phi_{ll} \end{bmatrix}$ , where  $\phi_{ij} = \Pr(s_t = j \mid s_{t-1} = i)$  with i, j = H, L and thus  $\sum_j \phi_{ij} = 1$  for  $\forall i$ .<sup>14</sup>

The monetary authority is assumed to inject money into the economy by making lump sum transfers to households:  $M_t - M_{t-1} = T_t$ , so as to control the nominal interest rate. The interest rate is set according to the Taylor rule:

$$i_t = r \pi_t^{*int} \left(\frac{\pi_t}{\pi_t^{*int}}\right)^{\gamma} \left(\frac{Y_t}{Y_{t-1}}\right)^{\theta} \exp(\epsilon_t), \tag{1}$$

where  $i_t$  is the gross riskfree nominal interest rate, r denotes the steady-state gross real interest rate,  $\pi_t$  is the gross inflation rate,  $Y_t$  is real output and  $\epsilon_t$  is the state dependent monetary policy shock which follows an i.i.d. white noise process in a given regime:  $\epsilon_t (s_t = i)$  $\sim N(0, \sigma_i^2), i = H, L.^{15}$ 

<sup>&</sup>lt;sup>14</sup>The model does not give any explanation for the choice of the long-run target in the sense that the long-run output level implied by  $\pi^{*L}$  is equal to the long-run output level under  $\pi^{*H}$ . The choice of the target might be explained by reasons related to political economy for instance which are exogenous to our model. For an empirical study of the reasons why the inflation target has changed over time see e.g. Ireland (2005).

 $<sup>^{15}</sup>$ We have defined the Taylor rule on output growth instead of the output gap in view of a future non-linear

The interest rate reacts to output growth and to the deviation of the inflation rate from the central bank's 'intermediate' inflation target,  $\pi_t^{*int}$ . The intermediate target is allowed to change progressively as a function of the central bank's long-run inflation target and the current inflation rate according to the rule:

$$\pi_t^{*int} = (\pi^* \left( s_t \right))^{1-\lambda} \left( \pi_t \right)^{\lambda}, \tag{2}$$

where the parameter  $\lambda$  is an inverse measure of the speed of convergence to the long-run target. A similar concept with a similar rule is used in Orphanides & Wilcox (1996). In contrast to the Markov-switching process implying discontinuous jumps in the long-run inflation target, the intermediate target and its convergence rule allow for less drastic changes in the monetary policy even after discrete breaks in regimes. Such smoothed changes seem to be more in line with empirically observable gradual shifts in the inflation target.<sup>16</sup> Other theoretical disinflation studies model changes in policy by a gradual exogenous linear decrease of the inflation target.<sup>17</sup> In contrast to these studies, our rule defines the operational intermediate target endogenously as a function of the central bank's long-run target and the current inflation rate which seems to better capture central banks' practice.

Note finally that we do not allow for changes in the policy rules' coefficients across regimes. Our definition of a regime is in line with empirical findings in e.g. Primiceri (2005) and Sims & Zha (2006). These authors emphasize the importance of changes in the volatility of the control error, as opposed to changes in the systematic monetary policy, in explaining changes in the business cycle.<sup>18</sup>

#### 3.2 The representative household

As is standard in New Keynesian literature, the representative household is maximizing its lifetime utility:

solution of the model. This modification can also be found in e.g. Erceg & Levin (2003) and Schorfheide (2005). It had a minor influence on our results. Our main conclusions especially were unaltered.

<sup>&</sup>lt;sup>16</sup>For some recent empirical studies on changes in the inflation target see e.g. Cogley & Sargent (2001 & 2005), Kozicki & Tinsley (2005) and Ireland (2005). A recent example for desired gradualism in policy shifts is provided by Greg Mankiw's (2006) letter to Ben Bernanke in which Mankiw suggests Bernanke to introduce an interest rate rule into monetary policy decision making, emphasizing that this change needs to be progressive. <sup>17</sup>See e.g. Ball (1995), Goodfriend & King (2005), Ireland (2005).

<sup>&</sup>lt;sup>18</sup>It should be noted that this view is debated in the literature: e.g. Clarida, Galí & Gertler (2000) have shown instead that the coefficients of the monetary policy rule have changed over time. For a theoretical model of regime shifts exploring changes in coefficients see Davig, Leeper & Chung (2004).

$$\max E_t \sum_{k=0}^{\infty} \beta^k \left[ \frac{C_{t+k}^{1-\sigma}}{1-\sigma} + b_L \frac{(1-L_{t+k})^{1-\chi}}{1-\chi} + b_m \frac{(M_{t+k}/P_{t+k})^{1-\eta}}{1-\eta} \right]$$

where  $M_t/P_t$  stands for real cash balances;  $C_t$  is a consumption bundle which is the CES aggregator of differentiated products:

$$C_t = \left[\int_0^1 c_t \left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}};$$

the parameter  $\varepsilon > 0$  is the elasticity of substitution between goods and  $c_t(i)$  stands for the household's demand for a differentiated good  $i \in [0, 1]$ :

$$c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} C_t.$$
(3)

Here,  $p_t(i)$  denotes the price set by firm *i*, and the aggregate price level can be expressed as:

$$P_t \equiv \left[\int_0^1 p_t \left(i\right)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

Labor hours supplied by the household to all firms are denoted by  $L_t = \int_0^1 l_t(i) di$ . The total of hours at the disposal of a household for labor and leisure is normalized to 1.

The maximization is subject to a sequence of period budget constraints:

$$P_t C_t + B_t + M_t \leqslant \int_0^1 \Pi_t (i) \, di + \int_0^1 w_t (i) \, l_t (i) \, di + i_{t-1} B_{t-1} + M_{t-1} + T_t,$$

with  $B_t$  denoting the end of period riskfree nominal bondholdings,  $M_t$  the end of period money holdings,  $\Pi_t(i)$  the profits of firm i,  $w_t(i)$  the nominal wage paid by firm i,  $T_t$  lump sum transfers and  $i_{t-1}$  the gross riskfree nominal interest rate paid on bonds as of the end of period t-1.

#### 3.3 Firms

There is a continuum of monopolistically competitive firms in the economy each of which produces a single differentiated good i. The production technology is linear in labor which is hired in a perfectly competitive labor market:

$$y_t\left(i\right) = Al_t\left(i\right)$$

Total factor productivity A is exogenous and will be held constant throughout the entire discussion.

Firms are price setters. We follow Calvo (1983) in assuming that each period firms face the probability  $\xi$  of being unable to reoptimize their prices. This probability is constant across firms and constant over time. In periods when a firm is not reoptimizing, it is assumed to index its previous period price  $p_{t-1}(i)$  by its previous period expectation of the intermediate inflation target  $E_{t-1}\pi_t^{*int}$ . The average of fixed prices is then

$$P_{t}^{fix} \equiv \left[ \int_{fix} \left( p_{t-1}\left(i\right) E_{t-1} \pi_{t}^{*int} \right)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} = P_{t-1} E_{t-1} \pi_{t}^{*int}.$$
(4)

The assumed indexation rule adapts the usual constant steady-state inflation indexation to an environment where the central bank's target is not constant, potentially unobserved and imperfectly anticipated by private firms. While the assumption may seem ad-hoc, it has several appealing features compared to the constant steady-state inflation indexation rule assumed in Schorfheide (2005).

First, the steady-state being defined as a non-stochastic state in which there is no uncertainty regarding the inflation target, the central bank's inflation target can deviate for a very long period from the steady-state inflation rate. Especially, the expected duration of a regime is plausibly much longer than the expected duration of newly set prices, which makes it inconsistent to assume that firms would use steady-state inflation rate as their index if they can have any information about the regime in place. In addition to being conceptually more appropriate, our indexation rule eliminates various analytical shortcomings linked to the steady-state indexation in a regime-switching setup. These will be discussed later.

Second, admitting that the Calvo price setting is a short-cut to more complicated price setting schemes, it should be noted that the  $E_{t-1}\pi_t^{*int}$  index can accommodate changes in firms' price setting behavior between high-target and low-target regimes. Such changes are broadly documented in empirical literature.<sup>19</sup> Indeed, the index reduces to a simple longrun target indexation when the type of the central bank is observable or learned by agents. Specifically, with  $\pi^{*L} = 0$  in the low-target regime, there will be no indexation when agents have learned about the regime they are in; in turn, in the high-target regime, agents index to  $\pi^{*H} > 0$ .

Finally, note that our model reduces to the steady-state indexation model in the steadystate.

<sup>19</sup>See e.g. Dotsey, King and Wolman (1999) or Gagnon (2006).



Firms allowed to reoptimize set their prices to maximize their future expected discounted flow of profits taking into account the probability of not being able to reoptimize in upcoming periods and considering the indexation followed in such periods:

$$\max_{p_t(i)} E_t \sum_{k=0}^{\infty} \xi^k Q_{t,t+k} \left[ p_t(i) \, X_{t,t+k} y_{t+k}(i) - TC \left( y_{t+k}(i) \right) \right], \tag{5}$$

where  $E_t Q_{t,t+k} = \beta^k E_t \left( \frac{u_c(C_{t+k})}{u_c(C_t)} \frac{P_t}{P_{t+k}} \right)$  is the nominal stochastic discount factor, TC(.) stands for total production costs and  $X_{t,t+k}$  is the cumulative index between the periods t and t+k:

$$X_{t,t+k} = \begin{cases} 1 & \text{if } k = 0\\ E_t(\pi_{t+1}^{*int}) E_{t+1}(\pi_{t+2}^{*int}) \times \dots \times E_{t+k-1}(\pi_{t+k}^{*int}) & \text{if } k > 0 \end{cases}$$

Denoting by  $P_t^*$  the average of reoptimized prices  $P_t^* \equiv \left[\int_{opt} p_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$ , the average price level can be expressed as:

$$P_t = \left[\xi(P_{t-1}E_{t-1}\pi_t^{*int})^{1-\varepsilon} + (1-\xi)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$
(6)

#### 3.4 Beliefs and expectations

Following Andolfatto & Gomme (2003) and Schorfheide (2005), we describe two solutions of this model. In the first version, agents are assumed to have *full information* about the current monetary regime. The j periods ahead expectation of a given variable  $z_t$  is:

$$E_t(z_{t+j}) = \left[ E(z_{t+j} \mid s_{t+j} = H) \quad E(z_{t+j} \mid s_{t+j} = L) \right] \Phi^j v_t$$
(7)

with

$$E\left(z_{t+j} \mid s_{t+j}\right) = \int_{-\infty}^{+\infty} z_{t+j}\left(\epsilon_{t+j}, s_{t+j}\right) f_{s_{t+j}}\left(\epsilon_{t+j}\right) d\epsilon_{t+j}$$

where  $v_t$  is a vector defined as follows:  $v_t = [1, 0]'$  if  $s_t = H$  and  $v_t = [0, 1]'$  if  $s_t = L$ .

In the second version, the monetary regime's type is unobserved. In this case, agents face a signal extraction problem. They will be assumed to make optimal inferences about the probability of each state conditional on all information available in the given period. The optimal inference is denoted by  $\hat{v}_t(\Omega_t) \equiv [\Pr(s_t = H \mid \Omega_t); \Pr(s_t = L \mid \Omega_t)]'$  where  $\Omega_t$  stands for the information set of private agents which contains all structural parameters of the model, all contemporaneous and past observable variables and the prior  $v_0$ . In this case, the j periods ahead expectation of a given variable  $z_t$  is given by

$$E_{t}(z_{t+j}) = \left[ E(z_{t+j} \mid s_{t+j} = H) \quad E(z_{t+j} \mid s_{t+j} = L) \right] \Phi^{j} \hat{v}_{t}(\Omega_{t}).$$
(8)

It should be noted, that no announcements about future states are assumed in the model. Therefore, even in the full information setting where the current type of the central bank is known, agents still remain uncertain about the future regime.

It is common in literature to interpret the unobservability of the regime as the monetary authority's lack of credibility.<sup>20</sup> At the same time, this credibility should not be confounded with the more standard credibility concept which is linked to time inconsistency and results from potential strategic interactions between an optimizing central bank and private agents. In our model, private agents' uncertainty about the type of the regime is exogenously assumed and may stem from sources different from low credibility such as the central bank's lack of transparency or inefficient communication about its objectives.<sup>21</sup>

#### 3.5 Equilibrium

Given the stochastic processes  $\{\epsilon_t, s_t\}_{t=0}^{\infty}$ , and given the initial state of the economy:  $P_0, M_0, Y_0$ and  $v_0$ , the equilibrium is described by a path of  $\{C_t, L_t, Y_t, M_t, P_t, W_t, i_t\}_{t=0}^{\infty}$  such that

- 1. Private agents make optimal inference about the probability of the state  $s_t$ ;
- 2. Households maximize their lifetime utility subject to constraints taking prices and wages as exogenous;
- 3. Firms set prices given their production technology and households' demand for their goods;
- 4. All markets clear and the transversality condition is satisfied.

#### 3.6 Parameter Set

The numerical simulation of the model uses the following benchmark parameters. We assume log-utility in all terms,  $\sigma = \chi = \eta = 1$ . The subjective quarterly time discount factor  $\beta$  is set to 0.99, which implies a steady-state annual real interest rate of about 4 percent. The elasticity of substitution between differentiated goods is  $\varepsilon = 11$ , which implies a steady-state mark-up of 10 percent. The probability of rule-of-thumb pricing is  $\xi = 0.75$  which corresponds to an average price duration of one year. The parameters of the Taylor rule are set to  $\gamma = 1.5$ ,  $\theta = 0.5$ . All these values are standard in New Keynesian literature.<sup>22</sup>

 $<sup>^{20}\</sup>mathrm{See}$ e.g. Erceg & Levin (2003), Andolfatto & Gomme (2003).

<sup>&</sup>lt;sup>21</sup>See e.g. Orphanides and Williams (2005).

<sup>&</sup>lt;sup>22</sup>See e.g. Galí (2003) and references therein.

Since we are interested in shifts between moderate and low inflation regimes, we set the long-run inflation target to an annual rate of 10 percent in the high-target regime and to 0 in the low-target regime. The standard deviation of the monetary policy shock is set to  $\sigma^{H} = 0.8\%$  (quarterly) in the high-target regime and  $\sigma^{L} = 0.25\%$  in the low-target regime. These values are broadly in line with empirical findings reported e.g. by Justiniano & Primiceri (2006) for the time-varying volatility of the U.S. monetary policy shock.<sup>23</sup> The learning mechanism crucially depends on the calibration of the regimes as captured by  $(\pi^{*i}, \sigma^{i})$ . We will therefore discuss the robustness of our results to the benchmark parameter set.

The parameters of the transition probabilities for remaining in the current state are set to  $\phi_{jj} = 0.96, j = H, L$ . This corresponds to Schorfheide's estimation for the U.S. economy and it implies an expected regime duration of 6.25 years. This may seem a bit short but, as discussed below, it does not influence our results too much.<sup>24</sup>

Finally, agents initial beliefs are set to the ergodic probabilities:  $v_{0H} = \frac{1-\phi_{ll}}{2-\phi_{hh}-\phi_{ll}}; v_{0L} = \frac{1-\phi_{hh}}{2-\phi_{hh}-\phi_{ll}}$ , and are hence both equal to 0.5.

#### 3.7 The Linearized Model

Following Schorfheide (2005), the model's optimality conditions are linearized around the non-stochastic steady-state defined as the state in which all shocks are zero, and there is no uncertainty about the type of the central bank. Steady-state inflation  $\pi^*$  is set to the long-run expected value of  $\pi^*(s_t)$ :  $\pi^* = \frac{1-\phi_{ll}}{2-\phi_{hh}-\phi_{ll}}\pi^{*H} + \frac{1-\phi_{hh}}{2-\phi_{hh}-\phi_{ll}}\pi^{*L}$ .

The first order approximation of the household's and firms' first order conditions and of the market clearing equations yields the following equation system.<sup>25</sup> The intertemporal consumption Euler equation takes the standard form of:

$$-\sigma \tilde{y}_t = \tilde{\imath}_t - E_t \tilde{\pi}_{t+1} - \sigma E_t \tilde{y}_{t+1} \tag{9}$$

The modified New Keynesian Phillips curve (NKPC) is described by the equation

$$\tilde{\pi}_{t} = E_{t-1}\tilde{\pi}_{t}^{*int} + \beta E_{t} \left( \tilde{\pi}_{t+1} - \tilde{\pi}_{t+1}^{*int} \right) + \frac{(1-\xi)(1-\beta\xi)}{\xi} \varsigma \tilde{y}_{t}.$$
(10)

 $<sup>^{23}</sup>$ Sims & Zha (2006) report even more marked differences between the monetary policy shock's volatility in alternative regimes. According to their estimate allowing for 9 different states, the monetary shocks' volatility was more than 12 times higher during what they identify as the 'Volcker regime' than during the 'Greenspan regime'.

 $<sup>^{24}</sup>$ Leeper & Zha (2003) assume that the high regime is maintained for about twelve and the low regime for about 25 years. Andolfatto & Gomme (2003) report estimates 8 to 10 years for Canadian data.

 $<sup>^{25}\</sup>mbox{Variables}$  with tilde denote percentage deviations from the steady state.

where  $\varsigma = \chi \frac{Y}{1-Y} + \sigma$  is the elasticity of real wage with respect to output in the steady-state. Relevant features of this Phillips curve will be discussed in the following subsection.

The linearized Taylor rule takes the following form:<sup>26</sup>

$$\tilde{\imath}_{t} = (\gamma + \lambda - \gamma \lambda) \,\tilde{\pi}_{t} + \theta \,(\tilde{y}_{t} - \tilde{y}_{t-1}) + \epsilon_{C,t}, \tag{11}$$

where  $\epsilon_{C,t}$  is a 'composite' monetary policy shock defined as:

$$\epsilon_{C,t} = (1 - \lambda) \left(1 - \gamma\right) \tilde{\pi}^*_{(s_t)} + \epsilon_t.$$

In both states, the composite shock follows a normal distribution, the mean and variance of which are conditional on the regime:  $\epsilon_{C,t} \sim N\left[(1-\lambda)(1-\gamma)\tilde{\pi}^*_{(s_t)};\sigma_{(s_t)}\right]$ .

The difference between the full information (FI) and the Bayesian learning (BL) settings is that in the FI setting, agents can distinguish between the two terms of the composite shock while in the BL setting they are assumed to observe  $\epsilon_{C,t}$  only, without knowing the precise value of the inflation target and the monetary policy shock. In this case, agents' optimal inference about the probability of being in state  $s_t = H$  takes the following form:<sup>27</sup>

$$\hat{v}_{1,t}\left(\Omega_{t}\right) = \frac{f_{H}\left(\epsilon_{C,t}\right)\left[\phi_{hh}\hat{v}_{t-1,1} + \phi_{lh}(1-\hat{v}_{t-1,1})\right]}{f_{H}\left(\epsilon_{C,t}\right)\left[\phi_{hh}\hat{v}_{t-1,1} + \phi_{lh}(1-\hat{v}_{t-1,1})\right] + f_{L}\left(\epsilon_{C,t}\right)\left[\phi_{hl}\hat{v}_{t-1,1} + \phi_{ll}(1-\hat{v}_{t-1,1})\right]};$$

the probability of state  $s_t = L$  is then  $\hat{v}_{2,t} = 1 - \hat{v}_{1,t}$ .

Here,  $f_i(\epsilon_{C,t})$ , i = H, L denotes the probability density function of the composite shock conditional on the regime  $s_t = 1, 2$ . This formula shows that the probability of a given state depends on agents' past inference and on the realization of the current composite shock given the probability distribution of the shock conditional on the states and given the perceived law of motion of the inflation target which is expressed by the transition probabilities.

Note that the inference problem is exogenous in the sense that it implies no feedback from agents' endogenous decisions to the inference. This assumption may be somewhat limiting the implied effect of learning in our model. However, the exogeneity of learning is crucial in order to keep the model tractable. It also allows us to solve the model consisting of equations (9),



<sup>&</sup>lt;sup>26</sup>Note that with the assumed policy rules, the Taylor principle requires  $\gamma + \lambda - \gamma \lambda > 1$ . Assuming values of  $\gamma > 1$ , this restriction requires  $\lambda < 1$ . This restriction is however not too restrictive, since  $\lambda = 1$  would imply that the central bank never converges to its target which is clearly implausible.

<sup>&</sup>lt;sup>27</sup>Strictly speaking, this is the value of the posterior probability distribution function for in a given point  $\epsilon_{C,t}$ . Note that the probability of observing this point in a continuous support is zero. This formula which is often used in the literature can however be considered as the limit of the probability of observing  $\epsilon_{C,t} \pm h$ , when  $h \to 0$ . For the deduction of the posterior probability distribution see e.g. Hamilton (1994) Ch.22.

(10) and (11) linearly using the method described in Sims (2002) and compute the nonlinear inference problem recursively. A detailed description of the solution can be found in Appendix A.

#### 3.8 Modified New Keynesian Phillips Curve

To understand the main characteristics of the modified New Keynesian Phillips Curve equation (10), it is useful to consider the following transformation of the equation:

$$\left(\tilde{\pi}_t - \tilde{\pi}_t^{*int}\right) = \left(E_{t-1}\tilde{\pi}_t^{*int} - \tilde{\pi}_t^{*int}\right) + \beta E_t \left(\tilde{\pi}_{t+1} - \tilde{\pi}_{t+1}^{*int}\right) + \frac{(1-\xi)(1-\beta\xi)}{\xi}\varsigma \tilde{y}_t.$$

To compare, the standard NKPC with constant steady-state inflation indexation is:

$$\tilde{\pi}_t = \beta E_t \tilde{\pi}_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi} \varsigma \tilde{y}_t;$$

and the Hybrid Phillips Curve (HPC) based on the assumption of lagged inflation indexation takes the form:<sup>28</sup>

$$\tilde{\pi}_t = \frac{1}{1+\beta} \tilde{\pi}_{t-1} + \frac{\beta}{1+\beta} E_t \tilde{\pi}_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi (1+\beta)} \varsigma \tilde{y}_t.$$

There are two important differences between the modified NKPC and the standard NKPC. First, the modified NKPC captures the inflation rate's fluctuations around the intermediate target while the standard NKPC describes the inflation rate's fluctuations around the constant steady-state. Second, in addition to the factors implied by the steady-state indexation, with the  $E_{t-1}\tilde{\pi}_t^{*int}$  indexation, unexpected changes in the target also influence the current inflation rate.

It is important to note however that the modified NKPC model does not introduce structural inflation persistence. When the intermediate target is fully expected one period in advance, i.e.  $E_{t-1}\tilde{\pi}_t^{*int} = \tilde{\pi}_t^{*int}$ , implied inflation fluctuations around the target are purely forward looking. Inflation persistence in our model stems from Bayesian learning combined with staggered price setting and the given monetary policy assumptions. This makes our model substantially different from the HPC in which the cyclical fluctuations around the steady-state inflation are structurally persistent: the current inflation rate depends on the past inflation rate as a result of the exogenously assumed backward looking behavior of a fraction of firms. The degree of inflation persistence implied by the HPC depends on the model's structural

 $<sup>^{28}</sup>$  For a description of these models see e.g. Woodford (2003, Chapter 3).

parameters and is thus constant. In the following sections, we will compare our model's predictions with those of the HPC model.

The above described features of the modified NKPC have important implications for our model's predictions regarding the real impact of monetary policy. In our model, only *unexpected changes in the target* imply real effects while the level of the target is neutral. In this respect, the modified NKPC is closer to New Classical theories which emphasize following Lucas (1972), that only unexpected monetary policy decisions can have an influence on real economic activity. In contrast, with the HPC *any change in the inflation rate*, and hence in the inflation target, is costly independent of whether it was expected by agents or not. This difference is due to the different sources of inflation persistence in the two models.

The standard NKPC on the other hand, has plainly counterfactual real implications in the regime-switching setup. With steady-state indexation, the level of the inflation target is predicted to affect the output gap if and as long as the target is different from the steadystate, while shifts in the target have no real costs. In particular, output is implied to be above (below) its steady-state as long as the target is above (below) steady-state inflation. In this model used by Schorfheide (2005), the real effects go beyond the adaptation of prices to the new long-run target after a regime switch, implying that expansionary monetary policy can generate higher output levels in the long-run. These implications are in contrast to conventional wisdom stressing the temporary nature of monetary policy's real effects. They also contradict empirically observed real costs of disinflations. Moreover, the quantitative model predictions crucially depend on the choice of the steady-state inflation, which is quite arbitrary in the current set-up. These counterfactual implications would also arise in regimeswitching models with partial backward looking indexation.<sup>29</sup>

Finally, it should be mentioned that the Markov-switching feature implies that the economy will always be relatively far from the point of approximation around which the model is linearized. Still, with the modified NKPC, the linearized model's dynamics were found to

<sup>&</sup>lt;sup>29</sup>Note that such short-comings do not only concern the linearized standard NKPC model. Long-run price dispersions arising in the non-linear solution imply long-run regime-conditional output to be below its steady-state in any case the long-run inflation target deviates from the constant steady-state index of fixed prices. The non-linear real predictions thereby also depend on the choice of the steady-state inflation rate. Moreover, the predictions of the linearized solution may well be of the opposite sign as those of the non-linear solution. The long-run real effects in the non-linear solution are comparable to the steady-state distortions of the Calvo model without indexation when the steady-state inflation is bigger than zero as discussed in e.g. Ascari (2004), Bakhshi et al. (2003) and Cogley & Sbordone (2005).

be reasonably close to those implied by the non-linear solution. Conversely, with the model building on steady-state indexation, the difference between the linear and non-linear impulse responses was found to be quite big.<sup>30</sup>

## 4 Disinflation

This section discusses the dynamic implications of our model in a disinflation experiment. Disinflation is defined as a shift from the high-target regime to the low-target regime.

#### 4.1 Disinflation experiment

In this subsection, we examine the theoretical response to a disinflationary policy based on the modified NKPC. Figure 2a displays the reaction of the inflation rate, the output gap, the realized inflation forecast error ( $FE_t = \pi_t - E_{t-1}\pi_t$ ) and agents's beliefs under full information (FI) and incomplete information with Bayesian learning (BL) after a regime switch from the high-target to the low-target regime in period t = 1 under quick convergence policy ( $\lambda = 0$ ).<sup>31</sup> In addition, Figure 2b illustrates the implications of arguably more realistic gradual shift of the operational target. This figure shows the impact of a regime shift under BL for different values of  $\lambda$ .

The following results stand out. First, in both the FI and the BL settings, disinflation is costly. Second, the disinflation scenario implied by the BL assumption seems to be more realistic in that it generates a slower decrease of the inflation rate, a more persistent decline of the output gap and in that it reproduces the observed persistent overprediction of inflation during the disinflation. The sacrifice ratio<sup>32</sup> of the disinflation policy computed for both settings shows in addition that disinflation is more costly when agents are uncertain about the type of the central bank they face  $(SR_{FI} = 0.52 \text{ vs. } SR_{BL} = 0.70).^{33}$  Last, the gradual

 $^{33}$ To compare, Ball (1994) and Erceg & Levin (2003) report a sacrifice ratio around 1.7 - 1.8% for the

<sup>&</sup>lt;sup>30</sup>A comprehensive discussion of the shortcomings of the non-linearities concerning the standard NKPC in a regime-switching model is beyond the scope of this paper. Non-linear solutions were found by discretizing the state space and using iteration on Euler equations (see Coleman, 1991). Result are available on request.

<sup>&</sup>lt;sup>31</sup>Disinflation paths in the BL setting are averaged over 1000 random draws of a sequence of monetary shocks (of 80 periods each)conditional on the regime. Simulations start from the steady state and the prior belief. Before the regime shift, 40 periods of  $s_t = H$  are simulated to allow agents to learn which regime they are in.

 $<sup>^{32}</sup>$ We follow Ball (1994) to compute the sacrifice as the undiscounted cumulated output gaps during the transition period divided by the change in the inflation rate. Output gaps were computed in deviation of the state-dependent long-run output level, which may slightly deviate from the steady-state in our model. We considered the 10 first periods after the regime shift.

change of the intermediate target reduces the decline of production while it slows down the disinflation process.

To understand what drives these results, note that the regime shift is unexpected in both the FI and the BL settings. In the BL setting however, the gradual learning of the new target keeps prices above the target for a longer period and thereby extends the period of output contraction and increases the sacrifice ratio.

The smoothing of the operational target ( $\lambda > 0$ ) affects both the initial unexpected impact of the regime-shift and the learning mechanism in the BL model. The gradual shift of the target decreases the unexpected change of the policy on one hand and thereby contributes to reduce real costs as indicated by a decrease in the sacrifice ratio (Figure 3). On the other hand gradualism decreases the observable difference between the two regimes. This then leads to more confusion and slower learning and thereby makes the disinflation process longer. It should be noted that the parameter  $\lambda$  increases inflation persistence only in interaction with the learning mechanism. In the FI setting, agents know the type of the monetary regime. Therefore all prices are adjusted to the new regime in the period following the shift. Inflation therefore always decreases quickly after a regime shift, independently of  $\lambda$ . In this setting, no trade-off arises between lower real costs and the speed of disinflation.<sup>34</sup>

The learning mechanism is not only determined by the parameter  $\lambda$  but also by the difference between the two regimes. Thus, a smaller difference between the high and the low inflation target or bigger volatilities of the monetary shock in either regime would have the same impact on the learning mechanism under incomplete information as the effect of an increase in  $\lambda$ .

In addition, the transition probabilities influence learning in a non-trivial way. While an increase in  $\phi_{hh}$  significantly slows down the disinflation process and increases its real costs, a similar increase in  $\phi_{ll}$  would have negligible effects only. This asymmetry is partly due to the difference between the control errors' volatilities under the two regimes and partly to the initial beliefs' role in learning.<sup>35</sup>

Volcker disinflation. There is however substantial uncertainty with regard to the precise value of observed sacrifice ratios. Cecchetti & Rich (2001) estimate sacrifice ratios for the U.S. in the range of 1.3 to 10. Their estimates are however not significantly different from 0.

<sup>&</sup>lt;sup>34</sup>Note that in the FI setting, the sacrifice ratio is also decreasing in  $\lambda$ . This is because the initial surprise change in the inflation target decreases when  $\lambda$  increases.

<sup>&</sup>lt;sup>35</sup>Changes in the transition probabilities also have a more subtle effect: since agents attribute a non-zero probability to a future regime shift, their inflation expectations will deviate from the long-run target even when they have learned about the regime. This tends to keep inflation slightly above (below) the target in the low

#### 4.2 Real costs and structural persistence

One of the major arguments in favor of the Hybrid Phillips Curve has been its capacity to explain real costs of disinflations.<sup>36</sup> To compare the implications of the modified NKPC with those of the structural inflation persistence model, we re-simulated the same disinflation experiment with the HPC (see Figure 4). As already discussed, the learning mechanism is exogenous in the sense that it only depends on the regime-conditional distribution of the monetary shocks and the parameters of the systematic monetary policy reaction function (see equation (3.7) and the definition of  $\epsilon_{C,t}$ ). Therefore, cross-model differences implied in the trajectories of economic variables are exclusively implied by differences in the price setting behavior which affect the transmission mechanism of monetary policy.

As can be seen in the figure, the disinflation trajectories implied by the modified NKPC under BL lie fairly close to those implied by the Hybrid Phillips Curve under both FI and BL. On one hand, this confirms that the HPC, is able to generate real costs and persistence of disinflation as well as to reproduce systematic inflation forecast errors.<sup>37</sup> On the other hand however, the results suggest that structural inflation persistence, as accounted for by the HPC, is not a necessary assumption to reproduce such results. Indeed, allowing for incomplete information with Bayesian learning can lead to very similar disinflation patterns without relying on structural inflation persistence.

## 5 Intra-regime policy interventions

As discussed in Leeper & Zha (2003), monetary policy shocks can affect the economy via two distinct channels. First, via the *direct effect* which is the effect of the shock conditional on the regime. This is captured by linear impulse response functions in standard models. In addition, there can be an *expectations-formation effect* when the monetary policy shock induces agents to change their beliefs about the regime in place. Shifts in beliefs in turn induce shifts in the decision rules as has been pointed out by Lucas (1976). Whether expectations-formation effects are important in practice is an empirical matter. If such effects are not important for typical policy interventions, conventional VAR models can be used to evaluate the impact

<sup>(</sup>high) target regime and has a real effect which is however negligible. When agents are more certain about the permanence of a regime (as indicated by higher values of  $\phi_{ii}$ , i = H, L), then these expectation effects are decreased. In the limit, when one of the states is absorbing ( $\phi_{ii} = 1$ ), the effect completely disappears. <sup>36</sup>See e.g. Fuhrer & Moore (1995) or Galí & Gertler (1999).

<sup>&</sup>lt;sup>37</sup>Note that while inflation forecast errors implied by the HPC are bigger than those implied in the modified NKPC model, such forecast errors do not play a role in generating real costs of disinflations in the HPC model.

of monetary policy. However, if expectations-formation effects are important, linear VARs cannot be used for this purpose.<sup>38</sup>

In this section, we discuss the effects of temporary monetary policy interventions modeled by temporary deviations from the systematic monetary policy rule captured by the Taylor rule. We first describe the direct and the expectations-formation effects and then study the relative importance of these effects.

In our model, the direct effect of a shock corresponds to impulse responses under full information. In contrast, under Bayesian Learning, impulse responses contain both direct and expectations-formation effects. Expectations-formation effects can therefore be computed as the difference between BL and FI impulse responses to a given shock.<sup>39</sup>

As is standard in the regime-shift literature, impulse responses will be defined as deviations from a variable's trajectory conditional on the state instead of being deviations from the steady-state.<sup>40</sup> A precise description of the computation of impulse responses can be found in Appendix B.

#### 5.1 Direct vs. Expectation-Formation Effects

Figure 5a displays the reaction of inflation, output, inflation forecast errors and the change in the inferred probability for state  $s_t = L$  as implied by the modified NKPC model under full information (dashed lines) and Bayesian learning (solid lines). The left-hand panels show impulse responses to an expansionary one standard deviation monetary policy shock  $\epsilon_1 = -\sigma_L$ , while the right-hand panels display responses to  $\epsilon_1 = -2\sigma_L$ , both with  $\lambda = 0$ . To compare, we show impulse responses implied by the HPC in Figure 5b.

The following results stand out.

First, under full information, the impulse responses implied by the modified NKPC display a fairly conventional pattern. This is because, as already discussed in section 3.8, when the monetary policy regime is fixed and observable, the modified NKPC collapses to the standard NKPC in deviations from the long-run target.<sup>41</sup>

Second, when agents cannot observe the type of the regime, the expectations-formation effect can modify the implied paths of impulse responses. The expectations-formation effect

 $<sup>^{38}{\</sup>rm See}$  Leeper & Zha (2003), Sims & Zha (2006).

 $<sup>^{39}\</sup>mathrm{See}$  Schorfheide (2005).

<sup>&</sup>lt;sup>40</sup>See e.g. Schorfheide (2005) or Davig, Leeper & Chung (2004) and references therein.

<sup>&</sup>lt;sup>41</sup>Note, that under  $\lambda > 0$ , the intermediate target can deviate from the long-run target in response to a shock. This would create a wedge between impulse responses implied by the modified and the standard NKPC. The difference is however minimal.

acts like a supply shock in the sense that when agents assign a higher (lower) probability to the high-target regime, the inflation rate would increase (decrease) and the output gap decrease (increase) relative to the FI response. Thereby, the expectations-formation effect tends to reinforce the shock's direct effect on the inflation rate and to counterbalance the direct effect on the output gap.

Third, in the BL setting, the bigger shock, by triggering the learning mechanism, increases the inflation forecast error more than proportionately. It also increases the forecast error's persistence (observe difference between BL and FI response to  $\epsilon_1 = -2\sigma_L$ ).

Finally, note that the impulse responses implied by the Hybrid Phillips Curve mainly differ from the modified NKPC model's impulse responses in the trajectory of the inflation rate. By the structural inflation persistence assumption, the inflation rate obviously becomes more persistent. This however does not change the nature of the expectation-formation effect neither does it influence too much the trajectories of the other variables.

#### 5.2 Incidence and size of expectations-formation effects

It is important to stress that even under the BL scenario, expectations-formation does not always play a significant role. Some shocks do barely trigger the learning mechanism, while others have a big impact on it. The incidence and the size of the expectations-formation effect implied by a monetary policy shock basically depends on the extent to which the shock induces agents to shift their beliefs about the type of the central bank.

Figure 6 shows the shift in agents beliefs in the period of the shock as a function of the shock's size for both states s = H, L, given the benchmark parameter set and assuming a quick convergence policy ( $\lambda = 0$ ). As can be seen, with the benchmark parameter set, the learning mechanism would essentially be triggered in the low-target regime for expansionary shocks which are bigger than one standard deviation or restrictive shocks bigger than two standard deviations. In contrast, in the high-target regime, the expectations-formation effect becomes most important for restrictive shocks between one and two standard deviations but overall much less important than in the low-target regime.

To understand the particular non-monotonous pattern in the shifts in beliefs after an intra-regime intervention, recall that the distribution of the composite shock  $\epsilon_{C,t}$  differs in both mean ( $\bar{\epsilon}_C^H < \bar{\epsilon}_C^L$ ) and variance ( $\sigma_H > \sigma_L$ ) under the different regimes. Since the high-target regime is associated with a high volatility of the disturbance, not only expansionary shocks but also unusually big restrictive shocks can be associated with this regime. This can

be seen in the figure by the increase in the probability of  $s_t = H$ , conditional on any of the two states, for bigger positive shocks.

Figure 7a shows 68% intervals for different variables' forecast error  $\tilde{x}_{t+j} - E_t(\tilde{x}_{t+j})$  under full information vs. Bayesian learning for both regimes  $s_t = H$  and  $L^{42}$  Figure 7b displays 90% intervals. The tighter forecast error intervals under FI and BL are very similar, confirming that, typically, shocks do not trigger the learning mechanism under the benchmark parameterization. At the same time, broader forecast error intervals reflect an important role of expectations-formation. Overall, these results indicate that learning can play a role in the propagation of some shocks but such shocks occur less frequently.<sup>43</sup>

The precise size of the expectations-formation effect is influenced by the calibration of the regimes and by the smoothing parameter of the intermediate target. Nevertheless, our results are in line with those reported by Leeper & Zha (2003) and Schorfheide (2005).

#### 5.3 Shocks in periods of transition

Our discussion so far has only focused on the implications of intra-regime interventions in stable regimes, i.e. when agents had enough time to learn about the regime they are in. This discussion necessarily ignored the impact of agents' *prior* beliefs on the propagation of a shock. This effect is however not negligible. The learning mechanism and, by the expectations-formation effect, the model economy's reaction to shocks are significantly influenced by prior beliefs. Agents' prior beliefs in turn, can be quite different in stable periods from their prior beliefs in periods of confusion about the regime, which would typically arise in transition to a new long-run target. Hence, the Bayesian learning model's impulse responses are substantially different in stable and in transition periods.

To illustrate, Figure 8 shows the impact of an expansionary shock of annualized 100 basis points ( $\epsilon_1 = -\sigma_L$ ) in a stable low-target regime as opposed to its effect when it occurs two periods after the start of the disinflation.<sup>44</sup> As can be seen, during the transition period when agents are less certain about the type of the central bank, the same shock has substantially



<sup>&</sup>lt;sup>42</sup>This corresponds to each variable's one standard error interval under FI. This needs no longer be the case under BL.

<sup>&</sup>lt;sup>43</sup>Note, that due to the nonlinearities of the BL impulse responses, the forecast error intervals of a given variable do not correspond to the intervals of impulse responses which were given to shocks of the same confidence interval. That is, typical responses lying within the forecast error interval of the variable may have been generated by atypical shocks and vice versa, atypical impulse responses may be generated by typical shocks.

<sup>&</sup>lt;sup>44</sup>For the stable regime, the shock occurs after 40 periods in this regime.

bigger impact on beliefs than in a stable low-target regime. This increases the shock's impact on the inflation rate and, after a temporarily higher output level, the shock leads to a persistently more pronounced decline in output than in the stable environment. Note also, that the inflation forecast error increases again as a result of greater regime uncertainty.

These results show that the regime-switch has a double impact in the short-run if the regime is not directly observable. First the shift in the long-run inflation target has a real effect until agents have learned about it. And second, it confuses agents. This significantly modifies the propagation of shocks during transition compared to stable periods because of the increased role of expectations-formation effects.

## 6 Regime-uncertainty and persistence

As discussed so far, the Bayesian learning model establishes a link between agents' uncertainty about the monetary policy regime and the propagation of monetary policy shocks in the economy. The mechanism is based on the varying incidence of expectations-formation effects depending on agents' prior beliefs about the monetary policy regime. On one hand, prior beliefs are determined by the entire history of monetary policy objectives and shocks and therefore reflect agents' clarity or confusion about the monetary policy regime. On the other hand, the role of learning increases when agents are confused. With learning being a major source of persistence in the model, inflation persistence is thereby directly linked to regime uncertainty and hence to the lack of transparency and stability of the monetary policy.

To measure the impact of regime-uncertainty on inflation persistence in our model, we computed the half-life of a shock conditional on agents' prior beliefs. Figure 9 displays the average half-life of inflation's impulse response to  $\epsilon_1 = -\sigma_L$  in  $s_t = L$  as a function of  $\hat{v}_{1,0}$ , agents' prior inference about being in the high-target regime in the period of the shock.<sup>45</sup> Since agents are in the low-target regime, a high probability given to the other regime reflects their confusion about the regime. The results confirm that inflation persistence increases with increasing regime-uncertainty. The average half-life of the shock is of 2.25 quarters when agents are not confused at all ( $\hat{v}_{1,0} = 0$ ). When agents initial belief is  $\hat{v}_{1,0} = 0.5$ , the number of periods necessary for the initial impact to halve increases by 50% to 3.37 quarters.

<sup>&</sup>lt;sup>45</sup>We thank an anonymous referee for the idea of this indicator. The half-life was computed based on the benchmark parameters with  $\lambda = 0.5$ , for each initial belief  $\hat{v}_{1,0} = 0, 0.1, ..., 1$ , for 1000 draws of stochastic shocks with a systematic 1 standard deviation expansionary shock in the second period. Half-lives were then averaged over the draws.

An alternative way to assess our model's performance is to compare its predictions for inflation persistence with empirically observed degrees of persistence. To this aim, we estimate the sum of autocorrelation coefficients  $\rho$  for the simulated theoretical inflation series using the same methodology as for the observed inflation series (see Section 2). Table 3 displays the estimated median unbiased  $\rho$  averaged over 1000 simulations for stable high-target (*H*) and low-target (*L*) periods as well as for a transition period from high to low target (*T*).<sup>46</sup> We also display the root mean squared inflation forecast error standardized by the shocks' state-conditional volatility. This indicator allows us to capture the impact of regime-uncertainty on inflation uncertainty while it abstracts from the effect of varying volatility across different periods. In addition, to compare our model with the one relying on structural persistence, we display the sum of autocorrelation coefficients implied by the Hybrid Phillips Curve model in Table 4. The following results are worth noting.

First, our model with Bayesian learning can generate realistic degrees of inflation persistence as captured by the sum of autocorrelation coefficients  $\rho$ . In particular, the persistence generated for stable periods matches well the values found for the U.S. for the second half of the 80s. Since the learning mechanism is typically very little triggered in stable periods, the degrees of persistence implied by the FI and the BL setups are very close in these periods. For the same reason, there is also little difference between the degrees of persistence implied for the stable high-target and the stable low-target regimes.

Second, the results confirm the link between regime-uncertainty and inflation persistence in the learning model. Note, that regime uncertainty also increases the uncertainty about future inflation as captured by the standardized root mean squared inflation forecast errors. Hence, our model correctly implies higher degrees of persistence for periods of greater uncertainty about future inflation. Again, the  $\rho$  generated by our BL model for the transition period is reasonably close to the degree of persistence observed in the U.S. in the 70s and in the

<sup>&</sup>lt;sup>46</sup>1000 draws of 80 periods each were simulated. Each simulation starts from the steady state in period 1. The regime switches to the high target in period 2 and to the low target in period 41. The stable high regime is defined between the periods 21 to 40 which allows agents 20 periods to learn previously. Transition periods are t = 41 to 60. This is longer than the effective learning period which introduces some downward bias into our persistence estimates. The period was chosen such as not to have too short a subsample. The stable L regime is t = 61 to 80. For each draw, we estimated the sum of autocorrelation coefficients for the given periods and then computed the average of the median unbiased estimator. Since the sample periods are rather short, the maximum laglength was set to 2 and the grid was created on the interval given by the sample persistence estimate plus and minus 3 OLS standard errors. The simulated series are generated with the benchmark parameters.  $\lambda = 0.5$ .

beginning of the 80s. Note, that uncertainty about the future inflation would also increase in the FI model after the regime shift.<sup>47</sup> However, the link between persistence and uncertainty is only established in the BL model, where uncertainty combined with unobservability of the current regime triggers the learning mechanism and thereby increases inflation persistence in unstable periods.

Finally, when comparing our model with the Hybrid Phillips Curve model, we find that the structural inflation persistence model can, by itself, generate relatively high degrees of persistence (see FI results). However, it needs the assumption of regime shift and learning to reproduce differences in persistence between stable and transition periods.<sup>48</sup> Also, the variation in the persistence implied by the learning mechanism with the HPC remains slightly below the change predicted using the modified NKPC. Overall, it should be stressed, that the assumption of structural inflation persistence is not necessary to generate realistic degrees of inflation persistence, and that it is not sufficient to track observed changes in inflation persistence. Both can in turn be reasonably well reproduced by regime shifts, imperfect information and Bayesian learning.

### 7 Conclusion

This paper presented an extended New Keynesian model in which the monetary policy regime is subject to shifts. The public cannot directly observe the type of the monetary regime. They instead perceive regime shifts as a stochastic process and use a Bayesian learning rule to make inference about the type of the regime. Firms' price setting is adapted to a world in which the inflation target of the central bank is not constant and not perfectly observable. The Phillips curve we suggest links the current inflation rate to agents' lagged expectations of the monetary policy's target in addition to expected future inflation rate.

Admittedly, our model is very simple. Future extensions might consider state-dependent price setting schemes and / or state-dependent monetary policy-rule coefficients both of which are arguably plausible in regime-switching environments.<sup>49</sup> It might also be interesting to define other objects of learning, such as potential output for instance.

<sup>&</sup>lt;sup>47</sup>This is due to the uncertainty about the future regimes and the unexpected nature of the regime shift.

 $<sup>^{48}</sup>$ The  $\rho$  implied by the HPC even seems to be too high, but it could be decreased by lower values of the Calvo parameter for instance. Note that partial indexation would also decrease inflation persistence. However, as already noted in section 3.8, partial indexation would imply counterfactual predictions for output in the regime-shift setup.

<sup>&</sup>lt;sup>49</sup>For a model allowing for state-dependent policy-rule coefficients see Davig, Leeper & Chung (2004).

Nevertheless, even at this stage, our model is able to reproduce realistic inflation and output dynamics both in transition periods and in stable regimes without relying on the assumption of intrinsic inflation persistence. In addition, the model presents a way in which monetary policy can shape inflation dynamics.

The mechanism by which the monetary policy's predictability influences inflation dynamics is driven by the varying incidence of learning in the model. Learning about the unobservable monetary policy is a major source of persistence in our model. The role of learning increases in periods of high uncertainty about the monetary policy regime thereby creating a link between inflation persistence and the stability and the transparency of monetary policy.

This endogenous mechanism is an alternative to the assumption of structural persistence in generating real costs of disinflations and inflation persistence. However, contrary to structural inflation persistence, our assumptions can in addition account for historically observed changes in the degree of inflation persistence.

The distinction between structural and non-structural inflation persistence is not only interesting for the comparison of inflation dynamics between different periods and / or different countries; it is also crucial for the optimal conduct of monetary policy. In this sense, our paper points to the importance of credibility and of the transparency in monetary policy communication.



## 8 Appendix A: Model Solution

The solution method used follows Sims (2002) and Schorfheide (2005).

The linear rational expectations model can be written in the following form:

$$\Gamma_0 \tilde{x}_t = C + \Gamma_1 \tilde{x}_{t-1} + \Psi \epsilon_{C,t} + \Pi \eta_t,$$

where  $\tilde{x}_t$  denotes the vector of the endogenous variables:  $\tilde{x}_t = (\tilde{y}_t, \tilde{\pi}_t, \tilde{i}_t, E_t \tilde{y}_{t+1}, E_t \tilde{\pi}_{t+1})$ ;  $\eta_t$ stands for expectational errors  $\eta_t = (\tilde{y}_t - E_{t-1} \tilde{y}_t, \tilde{\pi}_t - E_{t-1} \tilde{\pi}_t)$ . The only exogenous variable is the composite monetary policy shock  $\epsilon_{C,t}$ .

As shown in Sims(2002) the solution of such a system can be expressed as:

$$\tilde{x}_t = \Theta_1 \tilde{x}_{t-1} + \Theta_c + \Theta_0 \epsilon_{C,t} + \Theta_y \sum_{j=1}^{\infty} \Theta_f^{j-1} \Theta_z E_t \epsilon_{C,t+j}.$$

The solution for the matrices  $\Theta_1, \Theta_c, \Theta_0, \Theta_y, \Theta_f$  and  $\Theta_z$  can be found by Sims' gensys code.<sup>50</sup>

The model solution under full information (FI) and under Bayesian Learning (BL) differs only in the way expectations are formed. In the FI model agents can observe the monetary policy regime. FI expectations are defined by relation (7). Defining the vector  $\tilde{\pi}^{*\prime} \equiv [\tilde{\pi}^{*H}, \tilde{\pi}^{*L}]$ , the expected future composite shock in this setting is then:

$$E_t \epsilon_{C,t+j} = (1-\lambda) \left(1-\gamma\right) \tilde{\pi}^{*\prime} \Phi^j \upsilon_t.$$

In this case the model solution is a linear.

In contrast, in the BL scenario, where agents cannot directly observe the type of the central bank, the expectations are defined by (8) along with agents' optimal inference as expressed in (3.7). Therefore the expected future composite shock is:

$$E_t \epsilon_{C,t+j} = (1-\lambda) (1-\gamma) \,\tilde{\pi}^{*\prime} \Phi^j \hat{v}_t (\Omega_t) \,.$$

By the optimal inference  $\hat{v}_t(\Omega_t)$ , the BL model's solution becomes a function of the entire history of shocks and states as well as of agents' learning process and it can be highly nonlinear. The non-linear part is exogenous. The solution can be found recursively.

## 9 Appendix B: Computation of Impulse Responses

Due to non-linearities implied by the learning mechanism, impulse responses in the BL model are computed in the following way.<sup>51</sup> For both regimes  $s_t = H, L$ , at t = 0, the economy starts

<sup>&</sup>lt;sup>50</sup>Available at Chris Sims' website: http://sims.princeton.edu/yftp/gensys

 $<sup>^{51}</sup>$ For a similar simulation method see Schorfheide (2005).

off in steady-state and switches to the given regime  $s_t$  in period 1 where it remains fixed for the entire simulation. The period of the shock will be denoted  $t_0 + 1$ . For both states, two trajectories of the composite shock  $\left\{\epsilon_{C,t}^i\right\}_{t=1}^{t_0+h}$ , i = 1, 2 are constructed. The first sequence  $\left\{ \epsilon_{C,t}^1 \right\}_{t=1}^{t_0+h} \text{ is a random draw of the conditional distribution of the shock. The second sequence} \\ \left\{ \epsilon_{C,t}^2 \right\}_{t=1}^{t_0+h} \text{ is defined as follows: } \epsilon_{C,t}^2 = \epsilon_{C,t}^1 \text{ for } t \neq t_0 + 1 \text{ and } \epsilon_{C,t}^2 = \epsilon_{C,t}^1 + z\sigma_s \text{ for } t = t_0 + 1$ where s = H or L, and z is an integer scalar going from -3 to 3. For both shock sequences  $\left\{\epsilon_{C,t}^{i}\right\}_{t=1}^{t_{0}+h}$ , i = 1, 2, the trajectories of the inflation rate, the output gap, the nominal interest rate and the inferred probability of the regime type are computed. Denote by  $\{x_t^i\}_{t=1}^{t_0+h}$  the trajectory of a variable based on  $\left\{\epsilon_{C,t}^{i}\right\}_{t=1}^{t_0+h}$ , i = 1, 2. The impulse response of a variable to a given shock  $z\sigma_s$  is then defined as the average  $\left\{x_t^2 - x_t^1\right\}_{t=t_0}^{t_0+h}$  over 1000 random draws. For the computation of forecast error intervals,  $\left\{\epsilon_{C,t}^2\right\}_{t=1}^{t_0+h}$  is defined as follows:  $\epsilon_{C,t}^2 = \epsilon_{C,t}^1$  for  $t \neq t_0 + 1$  and  $\epsilon_{C,t}^2 = (1 - \lambda) (1 - \gamma) \tilde{\pi}^{*s}$ , s = H or L. Everything else is unchanged. The

impulse responses are then sorted across i = 1, ..., 1000 and the percentiles are then easily found in the sorted series.

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		1970Q1 - 1983Q4	1984Q1 - 1991Q3	1991Q4 - 2005Q1	1970Q1 - 2005Q1
CPI	mean	7.48	3.97	2.55	4.79
	$\sigma$	3.79	1.78	1.23	3.43
PGDP	mean	6.77	3.29	2.01	4.16
	$\sigma$	2.40	0.99	0.77	2.70
Forecast	RMSE	1.95	1.13	0.88	1.45

 Table 1: U.S. Inflation rate - descriptive statistics

Mean and standard error of inflation of the CPI and of the GDP deflator. Forecast: median one-quarterahead prediction for the growth of GDP deflator. Source: Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia. RMSE: root mean squared forecast error; forecast error is the difference between the forecast and the realized PGDP annualized qoq growth rate.

## Table 2: U.S. Inflation persistence

	1970Q1 to $1983Q4$	1984Q1 to $1991Q3$	1991Q4 to $2005Q1$	1970Q1 to 2005Q1
CPI	$\underset{(0.51;1.03)}{0.72}$	$\underset{(0.03;0.74)}{0.36}$	-0.19 (-0.51;0.16)	0.89 (0.79;1.02)
PGDP	$\underset{(0.55;1.02)}{0.74}$	$\underset{(0.11;0.79)}{0.44}$	$\underset{(-0.09;0.41)}{0.15}$	$\underset{(0.86;1.03)}{0.95}$

Hansen (1999) 'grid-bootstrap' median-unbiased estimates of  $\rho$ , the sum of auto-regressive coefficients of CPI and GDP deflator inflation; 90% confidence intervals in parentheses.

Lag-selection based on AIC. For technical details see Section 2.

Table 3: Inflation persister	nce in the	model:	modified	NKPC
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		stable H	Transition	stable L	Full sample
FI	Q	$\underset{(0.00;0.91)}{0.41}$	$\underset{(0.16;0.74)}{0.44}$	$\underset{(0.07;0.97)}{0.49}$	$\underset{(0.91;1.05)}{0.99}$
	$\frac{RMSE}{\sigma_s}$	0.69	0.98	0.70	0.84
BL	ρ	0.42 (0.00;0.93)	0.60 (0.42;0.78)	$\underset{(0.02;0.91)}{0.43}$	0.99 (0.91;1.05)
	$\frac{RMSE}{\sigma_s}$	0.73	1.01	0.75	0.89

Hansen (1999) 'grid-bootstrap' median-unbiased estimates of  $\rho$ , the sum of auto-regressive coefficients of simulated inflation under FI and BL; 90% confidence intervals in parentheses. Benchmark parameters,  $\lambda = 0.5$ . Lag-selection based on AIC. For technical details see Section 6.

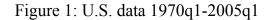
RMSE: root mean squared forecast error of inflation.

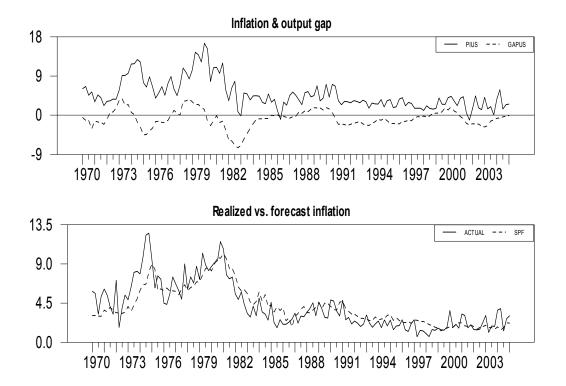
Table 4: Inflation persistence in the model: HPC

		stable H	Transition	stable L	Full sample
FI	ρ	$\underset{(0.29;1.10)}{0.69}$	$\underset{(0.66;0.76)}{0.71}$	$\underset{(0.33;1.12)}{0.73}$	$\underset{(0.97;1.02)}{1.00}$
BL	ρ	$\underset{(0.36;1.12)}{0.74}$	0.84 (0.76;0.90)	$\underset{(0.39;1.10)}{0.76}$	$\underset{(0.97;1.03)}{1.01}$

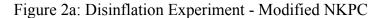
Hansen (1999) 'grid-bootstrap' median-unbiased estimates of  $\rho$ , the sum of auto-regressive coefficients of simulated inflation under FI and BL; 90% confidence intervals in parentheses. Benchmark parameters,  $\lambda = 0.5$ . Lag-selection based on AIC. For technical details see Section 6.

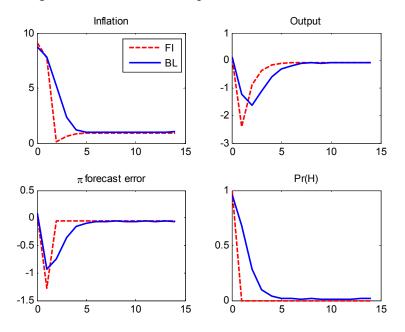




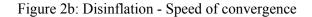


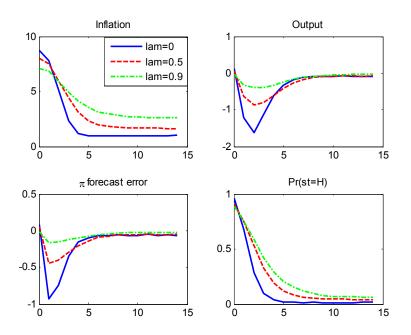
Inflation: qoq annualized percentage rates. Upper panel: CPI, Bottom panel GDP deflator. Source IMF IFS; Output gap source OECD; Inflation forecast: Survey of Professional Forecasters one quarter ahead median annualized growth rate of GDP price index (before 1996 GDP implicit deflator, before 1992 GNP deflator)





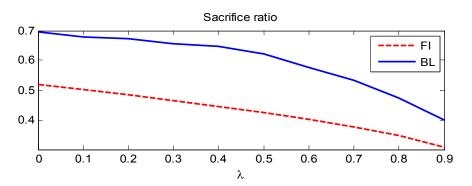
Regime shift in t=1; lamda=0; periods in quarters. Bayesian learning trajectories are averaged over 1000 random draws. Simulations start at t=-40 with prior beliefs are set to ergodic probabilities. Fourty periods of st=H are simulated before regime shift.



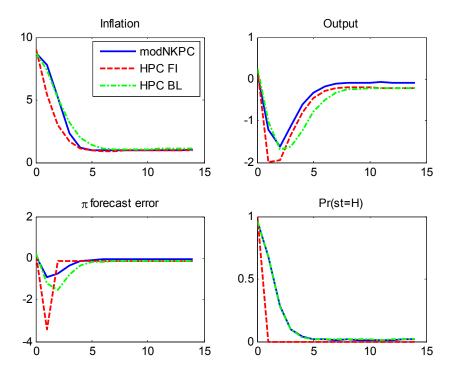


Disinflation trajectories under BL for different speeds of convergence (lamda). Disinflation simulations are averaged over 1000 random draws. Simulations start at t=-40 with prior beliefs set to ergodic probabilities.

Figure 3: Sacrifice Ratio



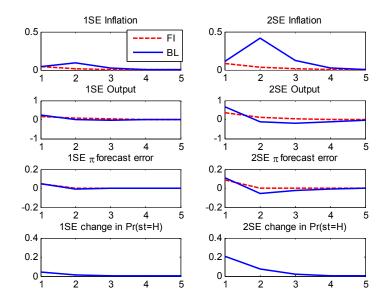
Sacrifice ratio under FI and BL as a function of the convergence parameter lambda.



## Figure 4: Disinflation - modified NKPC vs. HPC

Disinflation trajectories under BL assuming different pricing schemes. Disinflation simulations are averaged over 1000 random draws. Prior beliefs are set to ergodic probabilities. 40 periods of st=H are simulated before regime shift. Lambda=0.

Figure 5a: Impulse responses to  $\epsilon_1$  = - $\sigma_{st}$  and  $\ \epsilon_1$  = -2 $\sigma_{st}$ 



Impulse response of inflation, output, nominal interest rate and change in beliefs to  $\varepsilon = -\sigma_{st}$  (left-hand side) and to  $\varepsilon_1 = -2\sigma_{st}$  (right-hand side).  $s_t=L$ , lambda=0. Simulations averaged over 1000 random draws. Agents are allowed to learn about the regime for 40 periods.

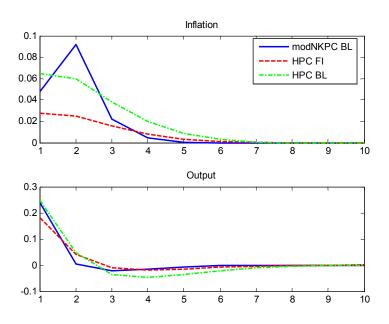


Figure 5b: Impulse responses - cross-model comparison

Impulse responses of inflation and output to a 1 standard deviation expansionary shock with modified NKPC and HPC.

Simulations averaged over 1000 random draws. Agents are allowed to learn about the regime for 40 periods.  $s_i=L$ , lambda=0.

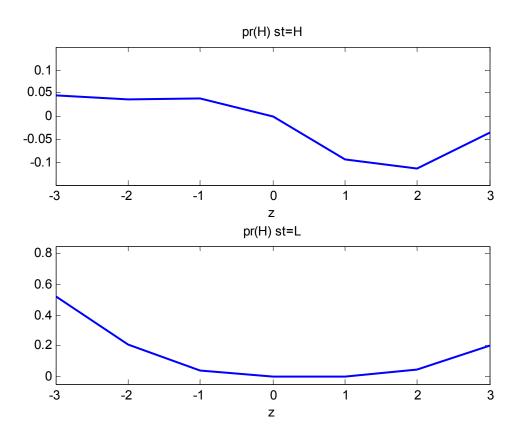
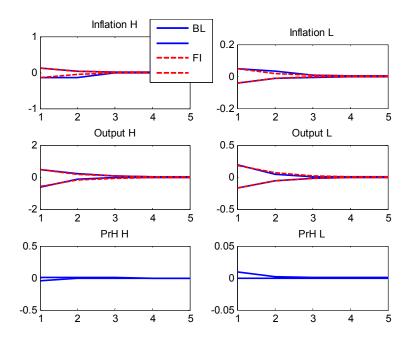


Figure 6: The role of expectations-formation

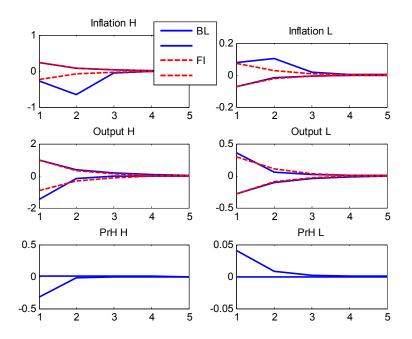
Change in the probability of state H as function of shock's size  $z\sigma_{st.}$  Values of z on horizontal axis.



## Figure 7a: Role of expectations-formation - 68% Forecast error intervals

68% probability intervals of forecast errors for inflation, output, nominal interest rate and change in beliefs. State=H (left-hand side); state=L (right-hand side); lambda=0. Intervals computed over 1000 random draws.

Figure 7b: Role of expectations-formation - 90% Forecast error intervals



90% probability intervals of forecast errors for inflation, output, nominal interest rate and change in beliefs. State=H (left-hand side); state=L (right-hand side); lambda=0. Intervals computed over 1000 random draws.



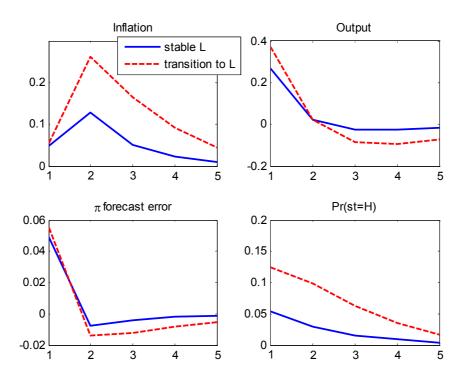
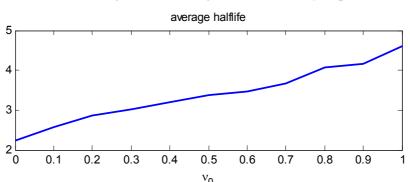


Figure 8: Impulse response - Stable regime vs. Transition

Impulse response to  $\varepsilon_1 = -\sigma_L$ . State  $s_t=L$ .  $\lambda=0.5$ . Stable L = after 40 periods in  $s_t=L$ ; Transition to L = 2 periods after regime switch from  $s_t=H$  to  $s_t=L$ .





 $v_0$ Half-life of initial impact of  $\varepsilon_1 = -\sigma_{st}$  shock on inflation as function of initial beliefs about high-target regime.  $s_t$ =L, lambda=0.5. Indicator averaged over 1000 random draws.

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