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Expecting the unexpected: economic growth under stress*

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Abstract

Large and unexpected moves in the factors underlying economic growth should be the main concern of policy makers aiming to strengthen the resilience of the economies. We propose measuring the effects of these extreme moves in the quantiles of the distribution of growth under stressed factors (GiS) and compare them with the popular Growth at Risk (GaR). In this comparison, we consider local and global macroeconomic and financial factors affecting US growth. We show that GaR underestimates the extreme and unexpected fall in growth produced by the COVID19 pandemic while GiS is much more accurate.

Growth vulnerability, Multi-level factor model, Stressed growth

C32, C55, E32, E44, F44, F47, O41

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1 Introduction

There is an increasingly growing literature on macroeconomic risk measurement that has recently become even more relevant because of the havoc generated by the global COVID19 pandemic. The significance of this literature stems from the necessity of policy makers to be prepared for extreme and unexpected shocks that generate severe recessions as well as for the implementation of corrective measures to minimize their risks; see Kilian and Manganelli (2008) and Alessi et al. (2014) for the importance of having appropriate measures of risk for policy makers and central banks, respectively. The high costs of crises underscore the need to strengthen the resilience of the economies, notably by assessing early on potential vulnerabilities that can lead to such costly events; see, the discussion in Rohn et al. (2015), who describe more than 70 vulnerability indicators that could be monitored to assess country risks in OECD economies, and Ludvigson, Ma and Ng (in press), who discuss the economic costs of the COVID19 pandemic in US.

Early proposals to measure growth risk were mainly based on macroeconomic uncertainty indexes; see, for example, Lahiri and Sheng (2010), who propose computing uncertainty based on the cross-sectional dispersion of survey-based forecasts and Rossi and Sekhposyan (2004), who propose an uncertainty index based on percentiles of the historical distribution of realized forecast errors. In a very influential work, Jurado, Ludvigson and Ng (2015) construct an uncertainty index based on factor augmented predictive regressions with conditionally heteroscedastic factors; see also Henzel and Rengel (2017) and Muntaz and Musso (in press) for uncertainty indexes based on factors extracted from large systems of predictors. The relation between uncertainty and growth has been investigated in a large number of works in the literature; see, among others, Baker et al. (2020), Baker, Bloom and Terry (2020) and Ludvigson, Ma and Ng (in press) and the references therein.

Although closely related, there is a relevant difference between growth risk and growth uncertainty as the former is only concerned with the left tail of the growth distribution and the latter with dispersion measures of forecast errors. This distinction is even more relevant when the distribution of growth is asymmetric, as it is often the case in times of deep crises; see, for instance, Breitung and Eickmeier (2015) and Plagborg-Möller et al. (2020) for evidence about asymmetries in economic growth fluctuations. As a result, several recent popular macroeconomic risk indexes are based on estimates of the full probability distribution of growth instead of focusing just on uncertainty; for an early contribution, see Ravazzolo and Rothman (2016), who propose a recession index indicator for US GDP based on growth density forecasts obtained as functions of oil prices. In a path-breaking contribution, Adrian, Boyarchenko and Giannone (2019a) propose the Growth at Risk (GaR) index that has become a very popular and influential macroeconomic risk index. At a given moment of time, the GaR is given by a quantile of the growth density, which is modeled as a function of “local” underlying financial factors. GaR has been adopted by the IMF as the main quantitative criterion to gauge global financial stability risk; see Prasad et al. (2019) and Adrian, Morsink and Schumaker (2020) for a description of the use of GaR at the IMF. The literature on GaR as a measure of growth risk is ample, diverse, and growing fast. Among the long list of works implementing GaR, we mention some to illustrate its influence. Alessandri, Del Vecchio and Meigietta (2019) and Buseti et al. (2020) compute GaR Italian growth, Delle Monache and Petrella (2020) obtain the conditional density of growth within a score driven framework instead of using quantiles, Ferrara, Mogliani and

Sahuc (2020) and Mazzi and Mitchell (2020) extend the GaR to the case of mixed frequency observations. They implement it in the European Union (EU) and the Euro area, respectively. Loria, Matthes and Zhang (2019) and Figueres and Jarocinski (2020) also compute the GaR in the Euro area and Plagborg-Möller et al. (2020) estimate GaR in the US and in other 13 advanced economies. Adrian et al. (2020) analyze the term structure of GaR based on a panel of 11 advanced economies. Finally, Brownlees and Souza (2019) show that GaR predictions based on quantile regressions are not more accurate than GaR predictions based on standard GARCH models.

Conceptually, the GaR mimics the spirit of the popular Value-at-Risk (VaR) measure in finance and, consequently, it also shares its caveats; see, for example, Haugh and Lacedelli (2020) for some weaknesses of VaR as a measure of financial risk. In particular, it is well known that the VaR is not designed to measure financial risk under stressed conditions; see Borio, Drehmann and Tsatsaronis (2014) and Flood and Korenko (2015) for very interesting discussions on stress testing in the area of finance. Stress testing, originated in engineering, is a technique to assess the stability of an object (e.g., type of material, a building, a machine) under adverse conditions. The object is subject to high levels of stress to evaluate its tolerance and identify its breaking point. In an economic context, stress testing refers to the analysis and forecasting of the left tail behaviour of the probability distribution of an economic measure, e.g. GDP growth, when hit by extreme financial and/or real shocks, which are abnormal and rare relative to those expected during tranquil times. As it happens with VaR, GaR is not designed to measure growth densities in stressed conditions of the underlying factors and, consequently, it could not be adequate to measure potential extreme risks that growth may be exposed to; see, for example, Plagborg-Möller et al. (2020), who show that the GaR did not yield useful advanced warnings of tail risk during the COVID19 pandemic. However, as mentioned above, large moves in the underlying factors should be the main concern of policy makers. To be prepared to face risky situations, as the economic havoc generated by the COVID19 pandemic, it is important to estimate growth densities under stressed factors. This is the proposal of González-Rivera, Maldonado and Ruiz (2019), who measure growth risk with the Growth in Stress (GiS) index defined as the minimum expected growth (or expected quantile of growth) when the underlying factors are stressed at a probability level. The factors considered by González-Rivera, Maldonado and Ruiz (2019) to compute the GiS were global macroeconomic factors. Using the GiS, policy makers have the opportunity of planning for the worse while hoping for the best. The GiS helps policy makers to deal with the trade-off between building greater resilience in normal times and reduce downside risk in highly stressed periods; see Adrian and Liang (2018) for a discussion of this trade-off. The first contribution of this paper is the comparison of GaR and GiS when measuring growth risk in extreme conditions in US GDP. We show that GiS is able to estimate the sharp decline in GDP growth observed during the COVID19 pandemic. Having a realistic measure of risk will contribute to the design of appropriate tools for strengthening the resilience of the macroeconomy in adverse conditions.

Whether risk should be measured by an estimated quantile of growth or a quantile estimated under stress, an equally important issue is the identification of the underlying factors that drive the estimation of the growth densities. Following Adrian, Boyarchenko and Giannone (2019a), growth densities are often estimated as a function of local financial factors.¹ The financial con-

¹The ability of financial factors to predict future real economic activity has been already discussed by Hatzius

ditions factors often used in empirical works are a summary of how easily firms, households and governments finance themselves. Usually, they are computed as a function of various asset prices and of the quantity and price of credit in the economy. The main argument for the link between financial factors and growth is based on financial prices incorporating market expectations of future price and output developments and, consequently, bearing timely information on future economic conditions. However, there is a debate in the literature about whether information relevant for assessing growth risk remains unaccounted for when risk estimation is based only on local financial indexes. Several authors investigate the dependence of the growth distribution on real macroeconomic variables; see, for example, Plagborg-Möller et al. (2020), Carriero, Clark and Marcelino (2020) and Reichlin, Ricco and Hasenzagl (2020). Whether the growth distribution depends on financial or macroeconomic factors, some others debate whether only local factors should be considered when assessing growth risk; see, for example, Mishkin (2011) and Breitung and Eickmeier (2015) for a discussion on the global character of some crisis, and Cerutti, Claessens and Rose (2019) on global financial factors. In general, potential shocks affecting growth can be either country-specific or of a global nature and, consequently, forecasting growth risk based on only “local” factors could be misleading in the current globalized world.

Our second contribution is to investigate which factors are more appropriate to estimate growth risk in US. We consider the full set of both “global” and “local” macroeconomic and financial factors. After estimating a multi-level Dynamic Factor Model (DFM) to jointly extract all factors, we show that both global and local macroeconomic and financial factors play an important role in measuring growth risk. We also conclude that, after taking into account global financial factors and macroeconomic factors, the local financial factors do not have any additional information to explain growth risk. Finally, armed with our empirical estimates of GiS for US GDP growth, we then turn to forecast the sharp growth decline during the COVID19 pandemic in early 2020. Based on a level of stress for the factors of 70%, we forecast a decline of about -10.01% for 2020Q2; the GaR forecast is much smaller -5.34%. The observed growth decline was -9.53% according to the International Monetary Fund (IMF).

The rest of the paper is organized as follows. In Section I, we describe the GaR, the GiS, and their differences. In Section II, we compare the GaR and GiS estimates of US GDP growth vulnerability in the context of the economic crisis triggered by the COVID19 pandemic. We also extract the underlying factors from fitting separate DFMs to systems of global and local macroeconomic conditions, and global and local financial variables. In Section III, we estimate the probability distribution of US GDP growth by using a combination of factors based on a multi-level DFM when all variables are jointly considered. Finally, in Section IV, we conclude and offer some final considerations.

et al. (2010), Matheson (2012), Giglio, Kelly and Pruitt (2016), De Nicoló and Luccetta (2017), Menden and Proaño (2017), Arrigoni, Bobasu and Venditti (2020) and Boyarchenko, Giannone and Kovner (2020), among others. The link between economic and financial conditions has experienced a revival after the global financial recession of 2008. As pointed out by Ng and Wright (2013), using US data from 1960 to 2012, all the post-1982 recessions have originated in financial markets, and these recessions are different from recessions where financial markets play a passive role.

2 Growth at Risk and Growth in Stress

We describe the estimation of the probability distribution of growth based on factor-augmented quantile regressions as well as the GaR and GiS measures of risk so that the paper is self-contained.

2.1 Forecasting the probability distribution of growth

Let GDP_t be the Gross Domestic Product (GDP) observed quarterly at time t , for $t = 1, \dots, T$. Define the corresponding quarterly growth as $y_t = \Delta \log(GDP_t)$. The one-step-ahead τ -quantile of the distribution of growth conditional on past growth and a set of underlying factors, $F_t = (F_{1t}, \dots, F_{rt})'$, is modeled by the following factor-augmented quantile-regression model²

$$q_\tau(y_{t+1}|y_t, F_t) = \mu(\tau) + \phi(\tau)y_t + \sum_{k=1}^r \beta_k(\tau)F_{kt}. \quad (1)$$

The factor-augmented quantile-regression model in equation (1) is appropriate for representing the potentially asymmetric and non-linear relationship between economic growth and the underlying factors. It has become very popular in the literature; see, Manzan (2005), Giglio, Kelly and Pruitt (2016), Adrian, Boyarchenko and Giannone (2019a) and González-Rivera, Maldonado and Ruiz (2019), among others. The parameters in equation (1) are estimated as proposed by Koenker and Bassett (1978) using the algorithm by Koenker and d'Orey (1987); see Ando and Tsay (2011) and Giglio, Kelly and Pruitt (2016) for the asymptotic properties. Finally, the goodness of fit of the estimated factor-augmented quantile regressions is estimated by $R^1(\tau) = 1 - \frac{\sum_{t=2}^T \hat{v}_t(\tau)[\tau I(\hat{v}_t(\tau) \geq 0) + (\tau-1)I(\hat{v}_t(\tau) < 0)]}{\sum_{t=1}^T y_t[\tau I(y_t \geq \bar{y}) + (\tau-1)I(y_t < \bar{y})]}$, where $\hat{v}_t(\tau) = y_t - \hat{\mu}(\tau) - \hat{\phi}(\tau)y_{t-1} - \sum_{k=1}^r \hat{\beta}_k(\tau)F_{kt-1}$ and \bar{y} is the sample mean of y_t ; see Koenker and Machado (1999). Note that $R^1(\tau)$ is the natural analog of the R^2 coefficient in a regression model.

The factors in (1) are usually extracted using Principal Components (PC)-based procedures from a set of N macroeconomic and/or financial variables of interest to explain the distribution of growth. For example, Adrian, Boyarchenko and Giannone (2019a) consider $r = 1$ factor extracted from a set of local financial variables.³ In another application, González-Rivera, Maldonado and Ruiz (2019) model the distribution of growth after extracting $r = 3$ factors from a set of international GDPs. Bai and Ng (2008) show that the PC estimated factors can be plugged in (1) as if they were observed as far as $\frac{T^{5/8}}{N} \rightarrow 0$ for $N, T \rightarrow \infty$. However, in applications for which N and/or T are not too large, we need to consider the uncertainty associated with the estimated factors; see Aastveit, Bjornland and Thorsrud (2016), Jackson et al. (2016) and Thorsrud (2020) for the importance of taking into account factor uncertainty in empirical applications. Confidence intervals for the factors can be constructed based on the asymptotic distribution derived by Bai (2003) under $N, T \rightarrow \infty$ with $\frac{\sqrt{N}}{T} \rightarrow 0$. However, when the temporal dimension T is not large enough relative to the cross-sectional dimension N , the

²For simplicity, we focus on the one-step-ahead growth densities. The extension to h -step-ahead densities is also of interest and it is left for further research; see Adrian et al. (2020) for the term structure of growth risk.

³In particular, they consider the Chicago Fed's National Conditions Index (NFCI), which provides a weekly update on US financial conditions in money markets, debt and equity markets and the traditional and "shadow" banking systems.

asymptotic distribution is not a good approximation. In this case, as proposed by Maldonado and Ruiz (in press), the asymptotic Mean Square Error (MSE) of the estimated factors can be modified using a subsampling procedure to incorporate parameter uncertainty and, as a result, to obtain more accurate confidence regions.

2.2 GaR and GiS

Adrian, Boyarchenko and Giannone (2019a) propose measuring the vulnerability of growth at time t by the GaR, defined as the quantile $\tau = 0.05$ of the conditional distribution of growth.⁴ Therefore, at each moment of time t , GaR_{t+1} is given by (1) with $\tau = 0.05$. It is important to remark that, although the GaR is an extreme left quantile of the distribution of growth, it is computed under “non-stressed” conditions, that is to say, the underlying factors are fixed at their estimated averages, \hat{F}_t . Our proposal is to measure growth risk when the factors themselves are under stress. The interesting question is to analyze the probabilistic distribution of growth when the factors are under extreme conditions. With this goal in mind, González-Rivera, Maldonado and Ruiz (2019) propose the GiS.

Suppose that we have estimated the quantile regression (1) for quantile τ . Since we also have the extracted factors F_t , we can calculate their multivariate probability density and, from that, we construct probability contours at a desired probability level α , say $\alpha = 95\%$ so that the contour is an ellipsoid that contains 95% of the values of F_t , leaving outside of the ellipsoid the most extreme 5% of the events. In this context, for a fixed quantile of growth τ , we define GiS as the minimum value of the expected τ -quantile of growth when the underlying factors are subject to α -probability extreme scenarios, that is,

$$GiS_{t+1} = \min q_\tau(y_{t+1}|y_t, F_t) \quad (2)$$

$$s.t. \quad g(F_t, \alpha) = 0,$$

where $q_\tau(y_{t+1}|y_t, F_t)$ is given by (1) and $g(F_t, \alpha) = 0$ is the α -probability contour, an ellipsoid that contains the true factor vector, F_t , with probability α . The values of F_t that are on the boundary of the ellipsoid $g(F_t, \alpha) = 0$ are considered the extreme events of the factors. As mentioned above, calculating the GiS in (2) requires to estimate not only the factor augmented quantile regression in (1) but also the multivariate probability density of the factors from which $g(F_t, \alpha) = 0$ is a probability contour. This multivariate density is obtained according to the asymptotic distribution of Bai (2003) modified by the subsampling procedure proposed by Maldonado and Ruiz (in press). For different levels of factor stress, i.e., different α , GiS calculates different values of the τ -quantile of the distribution of growth.⁵

To illustrate the differences between the GaR and GiS, let us examine the following example. We consider the following quantile regression model for $\tau = 0.05$ in which growth depends on two factors, F_{1t} and F_{2t} ,

$$q_{0.05}(y_{t+1}|F_t) = -3.5 - 0.7F_{1t} + 1.5F_{2t}. \quad (3)$$

⁴The GaR could also be defined for other forecast horizons and/or quantiles of the distribution of growth. In this paper, our focus is on one-step-ahead forecast horizon and the 5% quantile.

⁵By computing GiS for several quantiles, $q_\tau(y_{t+1}|y_t, F_t)$, $\tau = \tau_1, \dots, \tau_Q$, we obtain the full probability density of growth when some or all factors are stressed.

Figure 1 plots combinations of F_{1t} and F_{2t} for which we obtain the same value of the 5% quantile of growth; these are the iso-5%-quantile lines in red. Suppose that, at time t , the estimated factors are $\hat{F}_{1t} = 5$ and $\hat{F}_{2t} = 2$, which implies a GaR of -4. In Figure 1, we also plot the α -probability ellipses of the factors for $\alpha = 0.7$ and 0.95, in blue. The GiS is the tangency point between the α -ellipse and the iso-5%-quantile line. Thus, for $\alpha = 0.7$, the GiS is -8.5, and for $\alpha = 0.95$, the GiS is -11.5. These are big differences with $\text{GaR} = -4$. The reason being that GaR is calculated under “normal” circumstances, that is to say, when the factors are fixed at their estimated averages, which correspond to the central point of the ellipse in Figure 1.

By choosing different values of α , i.e. different levels of stress in the factors, the GiS provides an analysis of growth under different scenarios.⁶ For policy makers, knowledge of the growth density under stressed factors is a tool to assess whether or not the economy is too exposed to any of the factors and, if so, what actions to take in order to reduce exposure. In this sense, our GiS proposal to measure growth risk underscores the arguments in Breuer et al. (2009), who argue that measures based on historical experience, as the GaR, can lead to the risk of ignoring plausible but harmful scenarios, as those that we currently observe as a result of the COVID19 pandemic. By working with the probability contours for the underlying factors of growth, we provide a benchmark for plausibility and simultaneously, we choose the degree of severity of the stressed factors. The GiS captures plausibility by specifying how far we go into the tails of the factors’ distribution, while severity is maximized by systematically searching for the worst case (quantile of growth) in a factor region for a chosen level of plausibility; see also Flood and Korenko (2015) and Breuer et al. (2009) for discussions on the trade-off between plausibility and severity of stress scenarios.

3 Modeling growth risk in US: GaR versus GiS

We compare GaR and GiS estimates of US growth risk based on factors extracted from different sets of variables. First, we consider local and global financial factors. We extract the local factors from the same set of local financial variables used to construct the Chicago Fed’s National Conditions Index (NFCI). The global financial factors are those proposed by Arregui et al. (2018). Secondly, we consider local and global macroeconomic variables. The local macroeconomic factors are extracted from the database of McCracken and Ng (2016) and the global factors are extracted from a set of world growths as in González-Rivera, Maldonado and Ruiz (2019). For these four instances (local and global financial and local and global macroeconomic factors), we analyze and compare the performance of GaR and GiS estimates during the COVID19 pandemic.

⁶Scenario analysis is rather popular in the context of financial markets; see Glasserman, Kang and Kang (2015), who identify sensible combinations of stresses to multiple factors to assess financial risk; Hagfors et al. (2016) for scenario analysis of electricity prices in the context of quantile regressions; European Central Bank (2006) for the importance of scenario analysis in the context of stress testing in the financial sector, Rebonato (2019) for financial stress testing based on Bayesian nets, and Haugh and Lacedelli (2020) who carry out scenario analysis for derivative portfolios via DFMs. Finally, it is important to remark that the Basel Committee on Banking Supervision (2005) recommends choosing scenarios that are plausible and severe.

3.1 Financial factors

Adrian, Boyarchenko and Giannone (2019a) estimate US growth densities as functions of a local financial factor, in particular, the NFCI, which provides a weekly update on US financial conditions in money markets, debt and equity markets and the traditional and “shadow” banking systems; see Brave and Butters (2012) for a description of the NFCI. Positive values of the NFCI indicate financial conditions tighter than average, while negative values indicate financial conditions looser than average. We extract the latent local financial (LF) factors by PC based on the same variables underlying the construction of the NFCI with a cross-sectional dimension of $N = 105$ variables observed from 2005Q3 to 2020Q1 ($T = 59$ observations).⁷ Based on the determination criteria of Alessi, Barigozzi and Capasso (2010), the number of common LF factors is two. Consequently, we extract the two factors by PC. We note that the correlation between the first factor and the NFCI is 0.97.

Given the increasing globalization of the economy, we also consider the relationship between US growth and global financial (GF) factors as those proposed by Arregui et al. (2018) over the same period 2005Q3 to 2020Q1 with $N = 208$ variables. As before, the variables are standardized after correcting for outliers,⁸ and the Alessi, Barigozzi and Capasso (2010) criteria determines three factors.

For both cases, local and global financial factors, we compute the corresponding joint ellipses (2 factors) and joint ellipsoids (3 factors) with α coverage. In the upper panels (a) and (b) of Figure 2, we plot the 95%-ellipses and ellipsoids from 2005Q3 to 2020Q1, respectively. In blue, we show the ellipses corresponding to the Great Recession in 2008Q4 and 2009Q1, which are similarly dominated by a more disperse first factor moving to the right (tighter local financial conditions). In red, we show the ellipses corresponding to the pre-COVID19 quarter 2019Q4 and the beginning of the crisis in 2020Q1. It is interesting to note the sharp change in the shape of the ellipses from 2019Q4 to 2020Q1. At the beginning of the COVID19 crisis, 2020Q1, the dispersion of both factors is much larger than the dispersion in the pre-crisis quarter 2019Q4. Similarly, when we consider the three global financial factors, the blue ellipsoid (2009Q1) moves to the right, dominated by the dispersion of the first factor, and it has a very different shape from the red ellipsoid corresponding to the COVID19 crisis of 2020Q1.⁹ Overall, both (a) and (b) figures have similar profiles. Comparing the shapes of the blue and red ellipses and ellipsoids, we observe that the COVID19 crisis is quite different from the Great Recession, the latter characterized by a much larger increase in the uncertainty of the local and global financial conditions relative to the financial uncertainty observed in the COVID19 crisis.

After estimating the factors, we estimate the parameters of the factor-augmented quantile

⁷The NFCI is constructed on a weekly basis. Prior to factor extraction, we convert the weekly observations into quarterly frequency by averaging weekly observations within each quarter. For the attribution of weeks to overlapping quarters, we follow the same criteria as Adrian, Boyarchenko and Giannone (2019a). Weeks that start in one quarter and end in the next one are fully assigned to the latter quarter. After standardization, we also look for outliers using the procedure in Kristensen (2014). We find one outlier in the variable “T-note futures Euro/Dollar market depth” in 2008Q4.

⁸An outlier is found in Hungary 2015Q2, which may be due to the brokerage scandals during that year. A second outlier is found in Venezuela in 2018Q4, which may be attributed to large inflation and its repercussions in the stock market.

⁹The green ellipsoid in Figure 2 (b) corresponds to 2005Q3, which is a quarter of relative calm.

regressions for several quantiles of growth. In the upper panel of Table 1, we report the results for $\tau = 0.05, 0.50$ and 0.95 based on the local and global financial factors. As expected, the constant increases with the quantile τ and is always statistically significant. When we consider local financial factors, the two factors are statistically significant and negatively correlated with the quantiles $\tau = 0.05, 0.50$, so that tighter financial conditions (positive values of the factors) imply more risk. The local factors do not have any impact on the upper quantile $\tau = 0.95$. On the contrary, when we consider global financial factors, the three factors have major effects in the upper quantile $\tau = 0.95$, some effect in the median $\tau = 0.50$ but no effect in the lower quantile $\tau = 0.05$. The fit of the regression is the strongest in the lower quantile $\tau = 0.05$ with an R^1 of 36% for the local factors and of 34% for the global factors. A major conclusion is that the local and global financial factors explain different quantiles of growth. Based on the estimated quantiles, we estimate the corresponding growth densities through time, which are plotted in Figure 3 for the in-sample period from 2005Q3 up to 2020Q1. In the first column, the estimated growth densities are calculated based on the factors being centered at their means, and in the second column, the densities are based on the stressed factors at the 95%-ellipse-level. In the first column, we observe that, from 2005 to 2020, the growth densities overall tend to be more peaked and less disperse, which is consistent with a reduction in uncertainty over time, the exception being those densities around the Great Recession 2008-2010. In the second column, the densities reflect the potential risk that the economy may face when the factors are stressed at the 95% level. Naturally, the densities become flatter and shift their probability mass to the left; the left tails become thicker and so they provide the warning signals that are expected from any risk measure.

The estimation of the factor-augmented quantile regressions is sufficient to calculate the one-step-ahead GaR. In Figure 4 (a) and (b), we plot the 5%-GaR for 2005Q4 to 2020Q1 based on local and global financial variables respectively. In black, we also plot the actual growth over the same period. We observe that in both figures, the GaR is a very conservative measure and missed by a large margin the Great Recession 2008-2009 and the starting of the COVID19 crisis. When considering only the local financial factors, the one-step-ahead 5% GaR forecast corresponding to 2020Q2, reported in Table 2, was -0.61%; and when only global financial factors are considered, the one-step-ahead forecast of the 5%-GaR was -2.53%, which is much larger in magnitude than that based on local factors but still very short from the observed decline of -9.53%.

To compute GiS, we stress the factors at different levels. In Figure 4 (a) and (b), we also plot the one-step-ahead GiS for the 5% quantile of growth when the factors are stressed at the 70%, 95% and 99% levels. For local financial factors, we observe that GiS is much closer to the actual growth decline than GaR. The one-step-ahead GiS for 2020Q2 was -2.43%, -3.48% and -4.16% for $\alpha = 0.70, 0.95$ and 0.99 , respectively. For global financial factors, the respective values are somewhat similar with a 5% GiS for 2020Q2 of -3.51%, -3.88% and -4.12% for $\alpha = 0.70, 0.95$ and 0.99 , respectively; see Table 2.

In conclusion, if the growth densities are only driven by financial factors, the GaR provides a very conservative risk assessment even when we consider global financial factors. The GiS provides stronger warning signals but is still short from the actual growth. Hence, the need to explore additional factors.

3.2 Macroeconomic factors

Following a strand of the literature that claims that macroeconomic variables are better suited than financial variables to explain the growth distribution, we compute and compare the GaR and GiS based on local and global macroeconomic factors. We start by extracting local macroeconomic factors based on the popular database of McCracken and Ng (2016) with $N = 248$ variables observed from 2005Q2 to 2020Q1. After standardizing and correcting for outliers, the number of common underlying factors suggested by Alessi, Barigozzi and Capasso (2010) is two. We extract the global macroeconomic factors from a sample of annualized quarterly growths of $N = 63$ countries.¹⁰ The countries considered, which are listed in the online appendix, represent 91.62% of the world GDP in 2019, according to data by the World Bank.¹¹ We find the number of common factors is also two.

In the lower panels (c) and (d) of Figure 2, we plot the 95% ellipses of the estimated local and global macroeconomic factors. We observe the very different shapes of the ellipses corresponding to the Great Recession (in blue) and to the COVID19 crisis (in red), mainly in the case of local macroeconomic factors. Interestingly, when considering global macroeconomic factors, the ellipses corresponding to the COVID19 crisis show larger dispersion than those corresponding to the Great Recession crisis, and in both crises, the dispersion of the second factor is much larger than that of the first factor. Overall, when considering financial versus macroeconomic factors, the worldwide uncertainty is better captured by the global macro factors than by the financial factors.

In the lower panel of Table 1, we report the results of the factor-augmented quantile-regressions for $\tau = 0.05, 0.50$ and 0.95 based on the local and global macroeconomic factors. When considering local factors, the first factor is statistically significant in the three quantiles and is positively correlated with the quantile. The second factor is only relevant for $\tau = 0.05$. We also observe that the fit for $\tau = 0.05$ has greatly increased with a $R^1 = 39\%$, which is the largest fit in the four cases of Table 1. When considering global factors, the most notable difference with the local case is that the factors have not effect on the upper quantile $\tau = 0.95$. Putting in perspective all the results of Table 1, we conclude that we should expect improvements in risk assessment by combining financial and macro factors, mainly when our interest is in the left tail ($\tau = 0.05$) of the growth density.

In Figure 4 (c) and (d), we plot the 5%-GaR for the in-sample period 2005Q3 to 2020Q1 based on local and global macroeconomic variables respectively. We note that the one-step-ahead

¹⁰The GDPs have been obtained from the International Monetary Fund and the sample of countries has been chosen to maximize the amount of common data among countries. Note that in González-Rivera, Maldonado and Ruiz (2019), the factors are extracted from a panel of annual growths corresponding to 83 countries obtained from the World Bank database.

¹¹We also look for outliers using the procedure described by Kristensen (2014). We find two outliers in Thailand 2011Q4 and 2012Q1. These outliers may be due to the severe flooding occurred during the 2011 monsoon season, which caused the fourth costliest economic disaster according to the World Bank; see Tanonue et al. (2020). China 2020Q1 and Ireland 2015Q1 are also outliers. We think that the main reason for the outlier in China is that the COVID19 affected China one quarter before than the rest of the world. With respect to the large Irish GDP growth, it could be due to the relocation of intellectual property of a number of large multinational corporations, which was triggered by the Irish low corporate tax rates. Given the size of these companies, the boost to GDP growth was correspondingly large.

5%-GaR still provides a very conservative risk assessment while the 5%-GiS is sending the right signal about the extreme risk that is to come. The GaR based on macro factors is comparable to that based on financial factors. For the local macro factors, the one-step-ahead forecast of the 5%-GaR for 2020Q2 is -0.75%, very close to the -0.61% implied by the local financial factors; for the global macro factors, the GaR forecast is -2.83%, again very close to the -2.53% implied by the global financial factors. However, on computing the 5%-GiS with stressed factors, the one-step-ahead forecasts of the 5% quantile growth are almost twice as much as those based on financial factors. For instance, when considering global macro factors, the GiS forecasts are -6.56%, -8.69%, and -10.10% at the 70%, 95% and 99% levels, respectively. The actual decline in growth was -9.53%; see Table 2.

4 Obtaining the factors from a Multi-level Dynamic Factor Model

As shown in the previous section, both real and financial factors, either local or global, explain different quantiles of the distribution of growth. Though we have considered these factors separately, some authors have proposed modeling growth based on some combination of factors. For example, Çakmakli, Demrcan and Altug (forthcoming) propose a Markov-switching DFM for Turkey and choose local factors in an *ad hoc* manner assigning them to either “economic” or “financial” variables, but not to both. Other authors have considered growth models based on combinations of local and global economic factors; for example, the recent paper by Djogbenou (2020) proposes a two-level DFM with two specific developed and emerging economy activity factors in addition to a global economic factor.¹² However, these studies are restricted by the particular characterization of the variables from which the factors are extracted.

To jointly consider all factors and their ultimate joint effect on the US economic growth, we propose an innovative multi-level dynamic factor model that allows for common factors either to all the variables in the system, or to a subset of variables, or to a combination of subsets of variables. In the previous section, we considered the set of 624 variables separately. Now, we extract the factors using this entire set. We denote $X_t^* = (X_{1t}, X_{2t}, X_{3t}, X_{4t})'$, where X_{1t} and X_{2t} are the subsets of 105 and 208 variables used to construct the local and global financial factors, respectively, while X_{3t} and X_{4t} are the subsets of 63 and 248 variables considered to extract the local and global macroeconomic factors, respectively.

We follow Rodríguez-Caballero and Caporin (2019) to construct the multi-level DFM. We decompose the factor structure into different levels, with some factors associated with the full cross-section of variables (pervasive factors), some other factors impacting a specific subset of variables (non-pervasive factors), and other factors impacting several subsets of variables (semi-pervasive factors). As proposed by Hallin and Liska (2011), we determine the factor structure by analyzing the pairwise correlations between the factors extracted in the previous section.¹³ Accordingly, the final selection is 7 factors instead of the 9 originally estimated in the previous

¹²There are other proposals of models with global and local financial factors. However, as far as we know, these factors have not been related to economic growth; see Amiti, McGuire and Weinstein (2019) for a recent contribution.

¹³In the on-line appendix, we plot these correlations, the smooth histograms of each of the estimated factors, and the pairwise scatter plots.

section, and the final multi-level DFM selected is as follows

$$X_t^* = \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & \lambda_{13} & \lambda_{14} & 0 & 0 & 0 \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & 0 & \lambda_{25} & 0 & 0 \\ \lambda_{31} & 0 & 0 & \lambda_{34} & 0 & \lambda_{36} & 0 \\ \lambda_{41} & \lambda_{42} & 0 & 0 & 0 & 0 & \lambda_{47} \end{bmatrix} \begin{bmatrix} F_{1t}^* \\ F_{2t}^* \\ F_{3t}^* \\ F_{4t}^* \\ F_{5t}^* \\ F_{6t}^* \\ F_{7t}^* \end{bmatrix} + \varepsilon_t^*, \quad (4)$$

where F_{1t}^* is a pervasive factor that loads in all the variables in the system, F_{2t}^* , F_{3t}^* and F_{4t}^* are semi-pervasive factors with loadings in the global (financial and macroeconomic), financial (local and global), and local (financial and macroeconomic) variables, respectively. Finally, F_{5t}^* , F_{6t}^* and F_{7t}^* are non-pervasive factors that load on the global financial, local macroeconomic, and global macroeconomic variables, respectively. This factor structure explains the relation between the financial cycle and the business cycle, though both cycles have different characteristics; see Claessens, Kose and Terrones (2012).

Examining the structure of the multi-level DFM (4), we note that the local financial variables, X_{1t} , load on the common factor F_{3t} , which corresponds to the financial variables, and on the common factor F_{4t} , which corresponds to the local variables. However, there is not a non-pervasive separate factor for the local financial variables alone. Consequently, the information about the underlying local financial factors is already contained in the global financial and local macroeconomic variables. This result is in agreement with Reichlin, Ricco and Hasenzagl (2020), who conclude that the NFCI contains little advanced information on growth beyond what is already contained in the real economic indicators. Plagborg-Möller et al. (2020), estimating US growth risk, also conclude that the performance of a model with both a macroeconomic factor, extracted from the same data set of McCracken and Ng (2016), and a financial factor is indistinguishable from a model with only a macroeconomic factor.¹⁴ They show that financial variables contribute little to distributional forecasts, beyond the information contained in real indicators. In the same vein, Carriero, Clark and Marcelino (2020) find limited improvements in accuracy when using financial indicators in addition to macroeconomic indicators. Consequently, we simplify the model by considering only the variables in $X_t = (X_{2t}, X_{3t}, X_{4t})'$. Following the same methodology described above, we select the final multi-level DFM as follows

$$X_t = \begin{bmatrix} X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & 0 & 0 \\ \lambda_{21} & 0 & 0 & \lambda_{24} & 0 \\ \lambda_{31} & \lambda_{32} & 0 & 0 & \lambda_{35} \end{bmatrix} \begin{bmatrix} F_{1t} \\ F_{2t} \\ F_{3t} \\ F_{4t} \\ F_{5t} \end{bmatrix} + \varepsilon_t, \quad (5)$$

where, as above, F_{1t} and F_{2t} are the pervasive and semi-pervasive factors that load in all variables and in the global (financial and macroeconomic) variables of the system. The other three factors in (5) correspond to non-pervasive factors that only load in the global financial (F_{3t}), and local (F_{4t}), and global macroeconomic (F_{5t}) variables.

¹⁴Indeed, Plagborg-Möller et al. (2020) conclude that no predictors provide robust and precise advanced warnings about any features of GDP growth distribution other than the mean.

Estimation of model (5) is based on the procedure in Breitung and Eickmeier (2015) and Rodríguez-Caballero and Caporin (2019).¹⁵ In Figure 5, we plot the five factors extracted from the multi-level DFM (5) with their 95% confidence regions obtained as explained below. Neither the pervasive F_{1t} or the non-pervasive F_{3t} and F_{4t} factors warn about the plausibility of a forthcoming big decline in growth due to the COVID19 pandemic. However, the warning coming from the semi-pervasive second factor F_{2t} was strong and very strong from the non-pervasive fifth factor. It is this last factor F_{5t} that truly captures the recent sharp decline in the global macroeconomy.

After estimating the factors, we construct their confidence regions to obtain plausible stress scenarios. Even though there is not yet a formal result on the asymptotic distribution of the factors when they are extracted from multi-level models, we construct these regions based on the asymptotic distribution derived by Choi et al. (2018) for the pervasive factor extracted in the first step, which has the same asymptotic distribution derived by Bai (2003). For the rest of the factors, which are extracted based on the residuals from the previous step, we also assume asymptotic normality. Because they are based on residuals, their asymptotic MSE will be affected by parameter estimation uncertainty but this problem should be mitigated by extending the subsampling procedure of Maldonado and Ruiz (in press) to the multi-level framework. We obtain the joint confidence regions for the five estimated factors in (5) assuming asymptotic normality and implementing subsampling techniques to correct the asymptotic MSEs.

We estimate the corresponding factor-augmented quantile regression models for $\tau = 0.05$, 0.5 and 0.95 quantiles of growth. The estimated parameters are reported in Table 3. First, we observe that the autoregressive parameter is only statistically significant for the 5% and 95% quantiles but not for the median, and it is positive for the 5% quantile and negative for the 95% quantile. Consequently, when the current growth is positive, other things being equal, the inter-quartile range of the one-step-ahead distribution of growth narrows and the uncertainty decreases. This result is in agreement with empirical evidence on uncertainty rising in recessions; see Jurado, Ludvigson and Ng (2015) and Ludvigson, Ma and Ng (in press). Secondly, all factors are statistically significant for the 5% quantile and all but one for the 95% quantile, while for the median, only F_{3t} , the non-pervasive global financial factor, is significant. The pervasive factor F_{1t} and the semi-pervasive F_{2t} (global financial and macroeconomic variables) have both positive effects on both tails; on the contrary F_{3t} (global financial variables) has a negative effect on both tails and in the median, which implies that in good times, the one-step-ahead distribution of growth moves to the right, other things equal. The remaining non-pervasive factors F_{4t} (local macroeconomic variables) and F_{5t} (global macroeconomic variables) move in different directions when they affect either the 5% or the 95% quantiles. Thirdly, it is remarkable the increase in the measure of fit R^1 compared to those reported in Table 1 (partial DFMs). For the 5% quantile, the multi-level DFM fit is 49% compared to a range of 31-39% fit of the partial DFMs; and for the 95% quantile, the multi-level DFM fit is 36% compared to a range of 7-25% fit of the partial DFMs.

Based on the estimated quantiles of growth, we plot the estimated densities over time in Figure 6. In the left panel, the densities are calculated when factors are centered at their means.

¹⁵See also Aastveit, Bjornland and Thorsrud (2016), who propose an alternative estimation procedure for multi-level DFMs and a bootstrap procedure to construct confidence bounds for the factors.

The $\tau = 5\%$ quantile of these densities is the GaR value. In Figure 4 (e), we plot the GaR over time. The profile of this series is not very different from the GaR obtained with separate DFMs. Its magnitude is slightly larger than that of the GaR from separate DFMs, but it is still a rather conservative measure. In Table 2 (last column), we report the one-step-ahead forecasts of the 5% quantile based on the multi-level DFM. The one-step-ahead GaR for 2020Q2 is -5.34%, twice as much as the GaRs from separate DFMs with global factors, either financial or macro, but still far from of the actual decline in growth. GaR fell short as a realistic warning of the risk that the economy was facing at that time.

Finally, we construct the ellipsoids for the five factors to obtain plausible stress scenarios and calculate the GiS. We obtain the set of points on the surface of the ellipsoid with α -coverage that minimize the 5%-quantile growth by implementing the simple binary mesh algorithm proposed by Flood and Korenko (2015).¹⁶ For 70%, 95% and 99% levels of stress in the factors, the 5%-quantile GiS is computed by evaluating the factor-augmented quantile regression at the chosen points. In Figure 4 (e), we plot the corresponding 5%-quantile GiS over the sample period from 2005Q4 to 2020Q1. Note that in 2008Q4, the GiS was already warning of an extreme growth decline. The one-step-ahead GiS for 2020Q2 with stressed factors at the 70% level was -10.01, which is remarkably close to the observed growth decline of -9.53%. In the right panel of Figure 6, we show the growth densities calculated from the multi-level-factor-augmented quantile regression when the factors are stressed at the 95%-ellipsoid-level. This last picture summarizes our proposed tool for risk assessment. The policy maker has a total visualization of growth dynamics under stressed scenarios of her choice. Warning signals are coming from the quantiles in the left tail and the best scenarios under duress from the quantiles in the right tail.

5 Final considerations

We propose the GiS as a measure of growth risk when the economy is facing potential extreme events that are difficult to anticipate and, consequently, defy available econometric tools. In doing so, we answer two important questions. First, we analyze the nature of the underlying factors that drive the distribution of growth. We extract the factors from a multi-level DFM fitted to a large set of economic and financial variables collected both at the local and global level. We show the joint relevance of financial and macroeconomic factors. In addition, the fit of the factor-augmented quantile regressions to model growth is much higher at the extreme quantiles (e.g. $R^1 = 49\%$ for the 5% quantile) when we estimate a multi-level DFM than when we consider separate DFM based on subsets of variables. Secondly, we show how to choose severe and yet plausible stress scenarios based on the joint probability distribution of the underlying factors of the growth distribution. Our methodology allows the policy maker to choose the severity of the stress on the factors and construct the density of growth under these conditions. This is a visualization tool that permits to discover contingent exposures that could result in fatal economic losses. Applied systematically, the GiS is an useful and complementary tool for policy makers, who wish to carry out a multi-dimensional scenario analysis.

¹⁶Software is available in <https://cran.r-project.org/web/packages/SyScSelection/index.html>. In a spaced grid or mesh on the ellipsoid, the fineness parameter determines the number of points iterated along each dimension until the optimal combination of points is found. We choose a fineness parameter of 8. We have experimented with several values of the fineness parameter and our results are very robust to this choice.

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Table 1: Estimated quantile regressions for US growth. Parameter estimates of the factor-augmented quantile regressions ($\tau = 0.05, 0.50$ and 0.95) for the annualized quarterly GDP growth with p -values in parenthesis. In bold, those estimates that are statistically significant at the 10% level. R^1 is the goodness of fit coefficient.

| Financial Factors | | | | | | |
|-------------------|------------------------|------------------------|-----------------------|------------------------|------------------------|------------------------|
| | Local Factors | | | Global Factors | | |
| | $\tau = 0.05$ | $\tau = 0.5$ | $\tau = 0.95$ | $\tau = 0.05$ | $\tau = 0.5$ | $\tau = 0.95$ |
| μ | -1.69 (0.00) | 1.95 (0.00) | 4.59 (0.00) | -4.94 (0.00) | 2.07 (0.00) | 4.53 (0.00) |
| ϕ | 0.04 (0.79) | -0.13 (0.37) | -0.13 (0.77) | 1.17 (0.00) | -0.10 (0.59) | -0.14 (0.00) |
| β_1 | -3.18 (0.00) | -1.64 (0.00) | -1.17 (0.26) | -0.43 (0.53) | -0.87 (0.02) | -0.20 (0.00) |
| β_2 | -0.44 (0.10) | 0.51 (0.02) | 0.44 (0.54) | -0.20 (0.73) | -0.40 (0.20) | -0.88 (0.00) |
| β_3 | | | | -0.94 (0.11) | -0.51 (0.10) | -1.00 (0.00) |
| R^1 | 0.36 | 0.14 | 0.17 | 0.34 | 0.10 | 0.25 |

| Macroeconomic Factors | | | | | | |
|-----------------------|------------------------|-----------------------|-----------------------|------------------------|-----------------------|-----------------------|
| | Local Factors | | | Global Factors | | |
| | $\tau = 0.05$ | $\tau = 0.5$ | $\tau = 0.95$ | $\tau = 0.05$ | $\tau = 0.5$ | $\tau = 0.95$ |
| μ | -0.34 (0.29) | 2.15 (0.00) | 4.69 (0.00) | -4.36 (0.00) | 1.91 (0.00) | 4.77 (0.00) |
| ϕ | -0.61 (0.00) | -0.11 (0.39) | -0.24 (0.49) | 0.94 (0.00) | 0.04 (0.64) | -0.02 (0.94) |
| β_1 | 4.47 (0.00) | 1.22 (0.00) | 1.70 (0.04) | 0.25 (0.71) | 0.67 (0.00) | 0.94 (0.28) |
| β_2 | -0.36 (0.06) | -0.03 (0.86) | 0.11 (0.81) | 1.54 (0.00) | 0.00 (0.98) | -0.38 (0.52) |
| R^1 | 0.39 | 0.12 | 0.25 | 0.31 | 0.09 | 0.07 |

Table 2: US growth risk (in percentage over previous quarter) in 2020Q1 for 2020Q2. One-step-ahead forecasts of 5% quantiles of US growth computed by GaR (without stressing the underlying factors) and GiS (with factors stressed at 70%, 95% and 99%)

| | Financial Factors | | Macroeconomic Factors | | Multi-level factors |
|----------|-------------------|--------|-----------------------|--------|---------------------|
| | Local | Global | Local | Global | |
| GaR | -0.61 | -2.53 | -0.75 | -2.83 | -5.34 |
| GiS(70%) | -2.43 | -3.51 | -4.23 | -6.56 | -10.01 |
| GiS(95%) | -3.48 | -3.88 | -6.24 | -8.69 | -11.8 |
| GiS(99%) | -4.16 | -4.12 | -7.55 | -10.10 | -12.84 |

Table 3: Factor-augmented quantile regressions with factors estimated from the multi-level factor model. LS estimates of the parameters of the factor-augmented regression models for three quantiles of the US growth distribution, namely, 5%, 50% and 95%. $\beta_1, \beta_2, \beta_3, \beta_4$ and β_5 are the parameters of the total factor F_1 , global financial and macroeconomic factor F_2 , global financial F_3 , local macroeconomic F_4 , and global macroeconomic factor F_5 , respectively. p -values in parenthesis.

| | μ | ϕ | β_1 | β_2 | β_3 | β_4 | β_5 | R^1 |
|---------------|-----------------|-----------------|----------------|-----------------|-----------------|-----------------|-----------------|-------|
| $\tau = 0.05$ | -2.62 (0.00) | 0.15 (0.00) | 0.68 (0.00) | 2.19 (0.00) | -1.20 (0.01) | -1.21 (0.03) | 3.44 (0.00) | 0.49 |
| $\tau = 0.5$ | 2.04 (0.00) | -0.19 (0.37) | 0.45 (0.38) | -0.01 (0.99) | -0.87 (0.03) | 0.48 (0.29) | 0.58 (0.19) | 0.16 |
| $\tau = 0.95$ | 4.33 (0.00) | -0.24 (0.01) | 1.30 (0.00) | 0.62 (0.00) | -1.06 (0.00) | 0.23 (0.21) | -0.59 (0.00) | 0.36 |

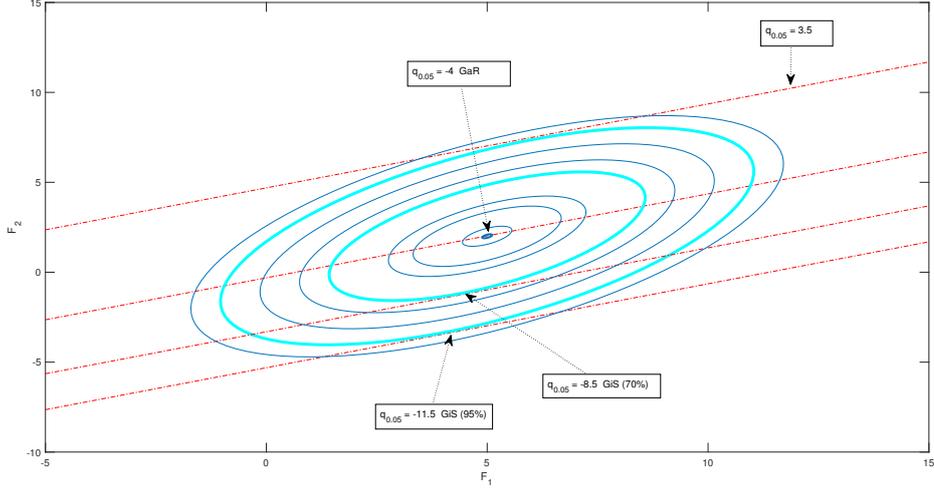
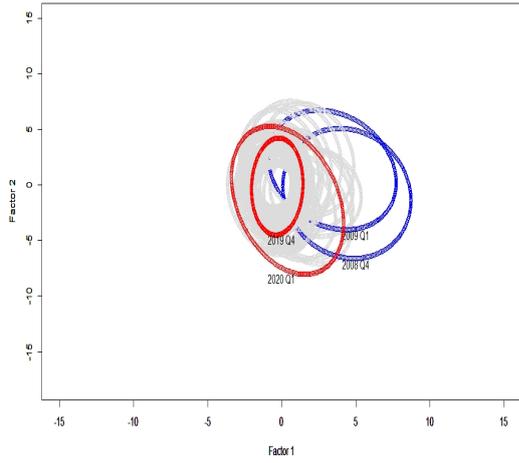
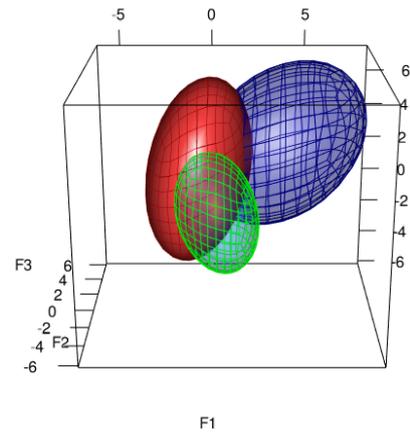


Figure 1: GaR and GiS.

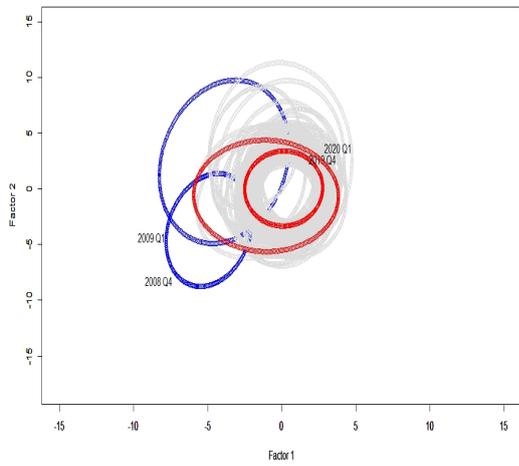
The red dash-dot lines are the iso-5% quantile growth lines for different values of the 5% quantile. There are four iso-quantile lines that correspond to $q_{0.05}(y_{t+1}|F_t) = 3.5, -4, -8.5$ and -11.5 . The blue ellipses are the contours of the bivariate probability density function of the factors. The highlighted blue ellipses are the 70% and 95% probability contours. The GaR is the value of the iso-5% quantile line at the center of the ellipse, which are the averages of the factors. The GiS is the value of the iso-5% quantile line that is tangent to the 70% probability contour (GiS = -8.5) or 95% probability contour (GiS = -11.5).



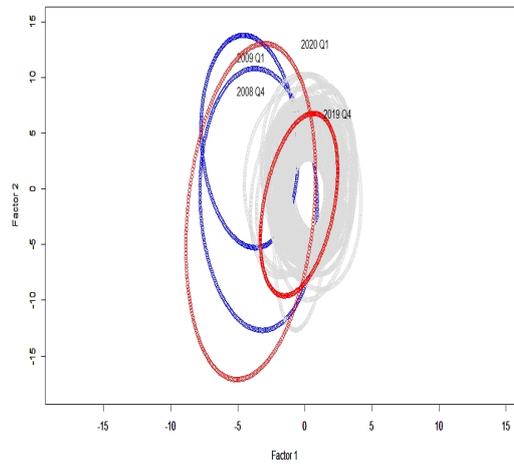
(a) Local financial factors



(b) Global financial factors



(c) Local macroeconomic factors



(d) Global macroeconomic factors

Figure 2: 95% ellipses and ellipsoids for different combination of factors.

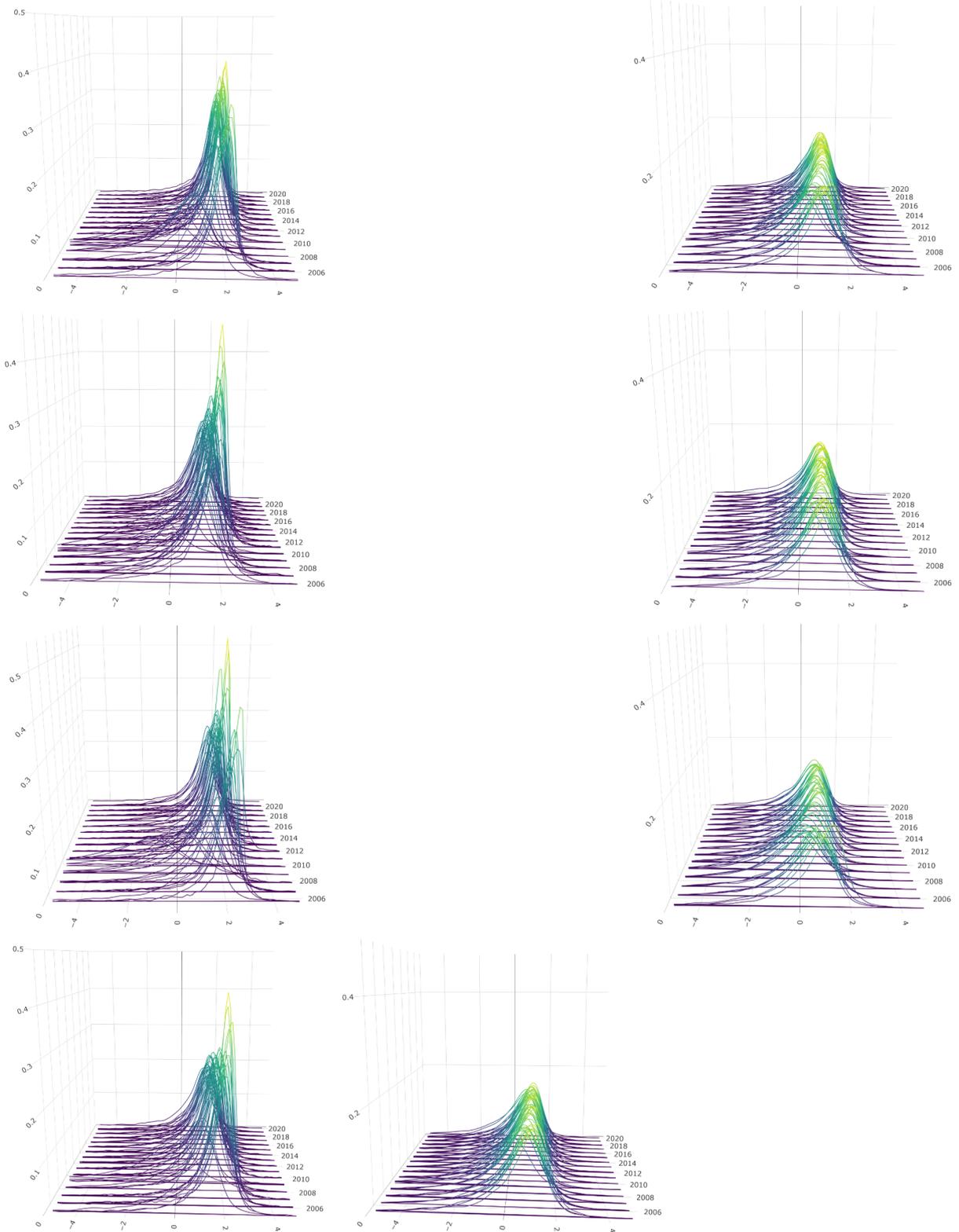


Figure 3: Growth densities obtained from different factor-augmented quantile regressions. The first row corresponds to the densities based on local financial factors; in the second row, densities based on global financial factors; in the third row, densities based on local macroeconomic factors; and in the bottom row, densities based on the global macroeconomic factors. In the first column, densities are obtained with the factors centered at their means, while in the second column, the densities are obtained with the factors stressed at the 95%-ellipsoid-level.

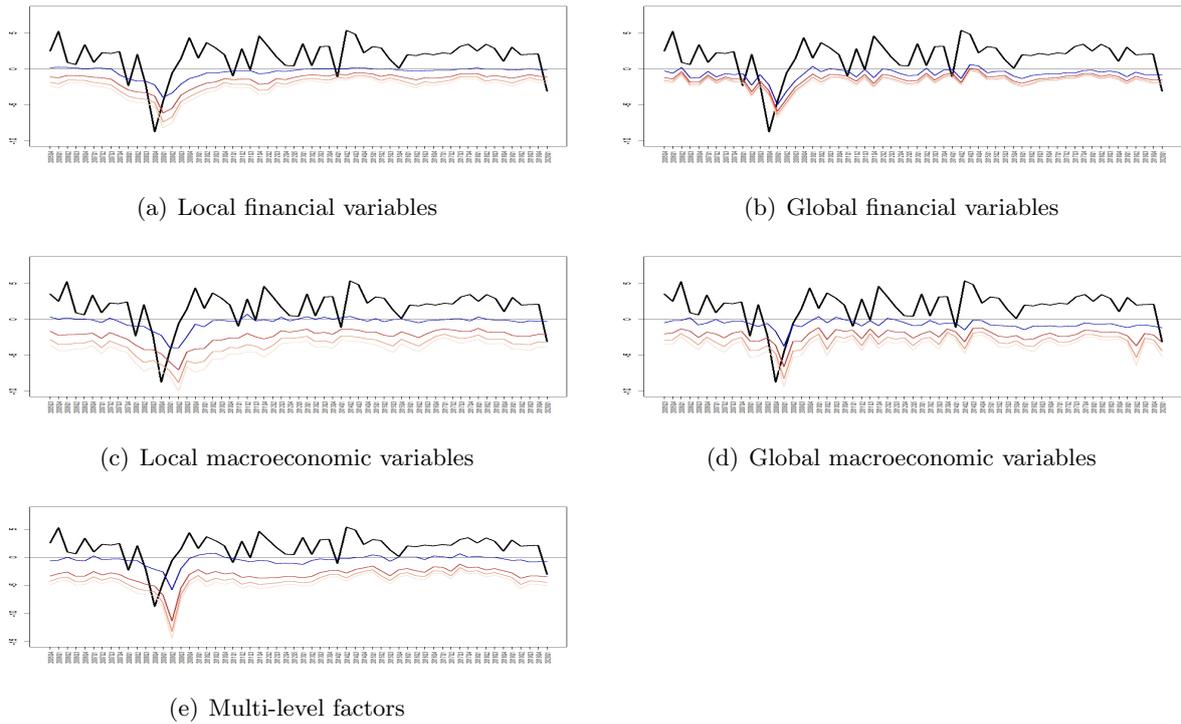


Figure 4: Actual Growth and risk measures obtained from different factor-augmented quantile regressions. In each panel, the black line is the actual growth; the blue line is the 5%-GaR; the sequential red palette lines (from high to low intensity) are the 5%-quantile GiS for 70%, 95% and 99% stress levels in the factors.

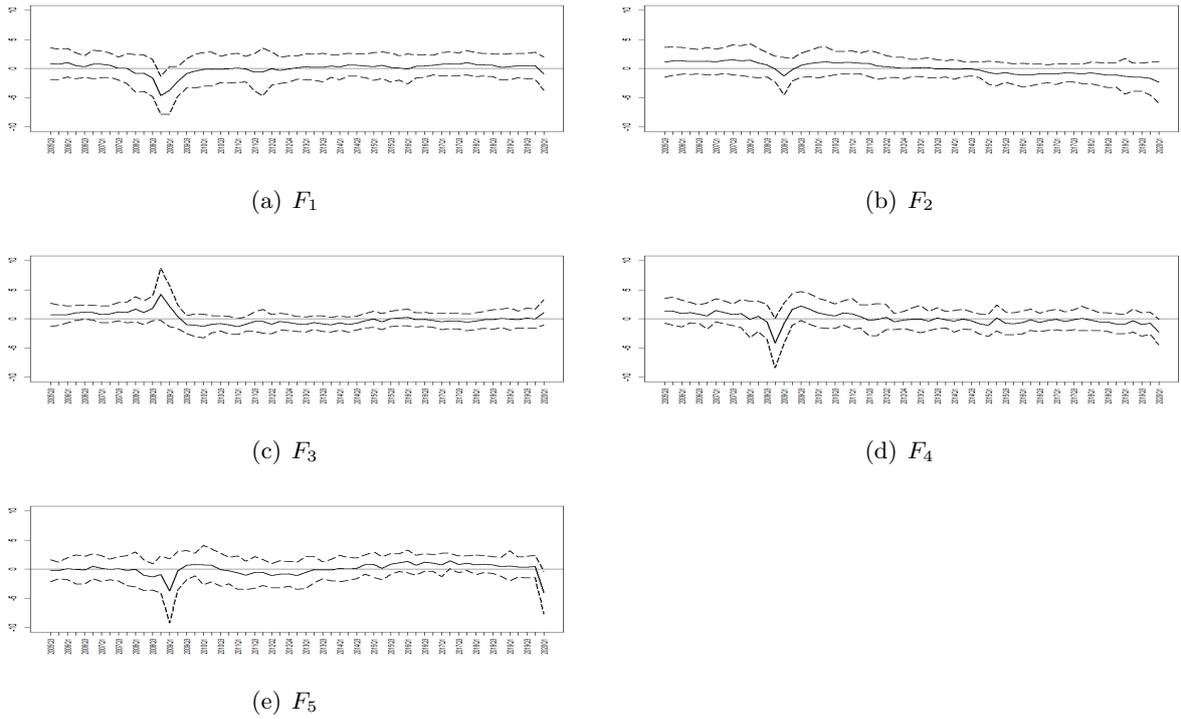


Figure 5: Factors estimated from the multi-level DFM with their 95% confidence bounds.

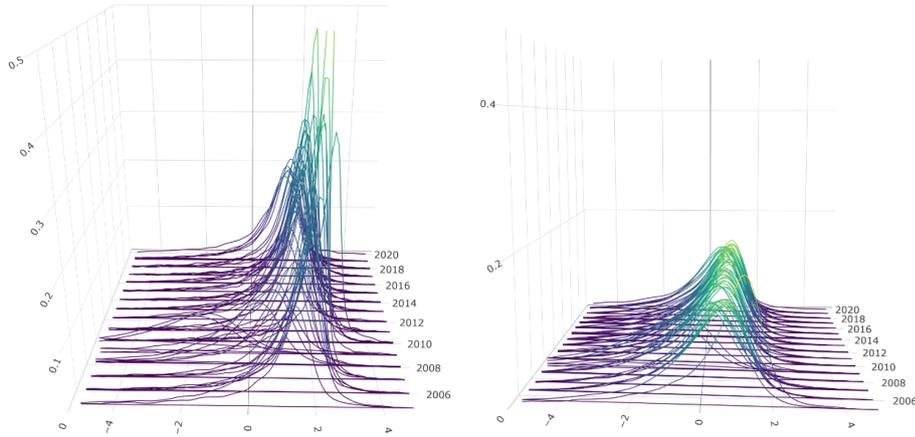


Figure 6: Growth densities estimated from the factor-augmented quantile regression model based on multi-level factors.

In the left panel, the densities are calculated when factors are centered at their means, while in the right panel, the densities are obtained when the factors are stressed at the 95%-ellipsoid-level.

Appendix: To be published online

Countries considered to construct the global growth factors.

Table 4: Countries considered to compute the global macroeconomic factors.

| | Country | GDP (Percentage over world GDP) | | Country | GDP (Percentage over world GDP) |
|----|----------------|---------------------------------|----|-----------------|---------------------------------|
| 1 | South Africa | 351,431.65 (0.40%) | 33 | Denmark | 348,078.02 (0.40%) |
| 2 | Argentina | 449,663.45 (0.51%) | 34 | Estonia | 31,386.95 (0.04%) |
| 3 | Brazil | 1,839,759.04 (2.10%) | 35 | Finland | 268,761.20 (0.31%) |
| 4 | Canada | 1,736,425.63 (1.98%) | 36 | France | 2,715,518.27 (3.10%) |
| 5 | Chile | 282,318.16 (0.31%) | 37 | Germany | 3,845,630.03 (4.40%) |
| 6 | Colombia | 323,802.81 (0.37%) | 38 | Greece | 209,852.76 (0.24%) |
| 7 | Mexico | 1,258,286.72 (1.43%) | 39 | Hungary | 160,967.16 (0.18%) |
| 8 | Peru | 226,848.05 (0.26%) | 40 | Iceland | 24,188.04 (0.03%) |
| 9 | United States | 21,374,418.88 (24.37%) | 41 | Ireland | 388,698.71 (0.44%) |
| 10 | Venezuela | 482,359.32 (0.55%) | 42 | Italy | 2,001,244.39 (2.28%) |
| 11 | China | 14,342,902.84 (16.35%) | 43 | Latvia | 34,117.20 (0.04%) |
| 12 | Hong Kong | 366,029.56 (0.42%) | 44 | Lithuania | 54,219.32 (0.06%) |
| 13 | India | 2,875,142.31 (3.28%) | 45 | Luxemburg | 71,104.92 (0.08%) |
| 14 | Indonesia | 1,119,190.78 (1.28%) | 46 | Malta | 14,786.16 (0.02%) |
| 15 | Israel | 395,098.67 (0.45%) | 47 | Moldova | 11,955.44 (0.01%) |
| 16 | Japan | 5,081,769.54 (5.79%) | 48 | Netherlands | 909,070.40 (1.04%) |
| 17 | Jordan | 43,743.66 (0.05%) | 49 | Norway | 403,336.36 (0.46%) |
| 18 | Kazakistan | 180,161.74 (0.21%) | 50 | Poland | 592,164.40 (0.68%) |
| 19 | Korea, Rep. | 1,642,383.22 (1.87%) | 51 | Portugal | 237,686.08 (0.27%) |
| 20 | Malaysia | 364,701.52 (0.42%) | 52 | Romania | 250,077.44 (0.29%) |
| 21 | Philippines | 376,795.51 (0.43%) | 53 | Russia | 1,699,876.58 (1.94%) |
| 22 | Singapore | 372,062.53 (0.42%) | 54 | Serbia | 51,409.17 (0.06%) |
| 23 | Thailand | 543,649.98 (0.62%) | 55 | Slovenia | 53,742.16 (0.06%) |
| 24 | Taiwan | 586,000 (0.67%) | 56 | Slovak Republic | 105,422.30 (0.12%) |
| 25 | Vietnam | 261,921.24 (0.30%) | 57 | Spain | 1,394,116.31 (1.59%) |
| 26 | Turkey | 754,411.71 (0.86%) | 58 | Sweden | 530,832.91 (0.61%) |
| 27 | Austria | 446,314.74 (0.51%) | 59 | Switzerland | 703,082.44 (0.80%) |
| 28 | Belarus | 63,080.46 (0.07%) | 60 | Ukraine | 153,781.07 (0.18%) |
| 29 | Belgium | 529,606.71 (0.60%) | 61 | United Kingdom | 2,827,113.18 (3.22%) |
| 30 | Czech Republic | 246,489.25 (0.28%) | 62 | Australia | 1,392,680 (1.59%) |
| 31 | Croatia | 60,415.55 (0.07%) | 63 | New Zealand | 206,928.77 (0.24%) |
| 32 | Cyprus | 24,564.65 (0.03%) | | | |

The Table reports the countries used to extract the global macroeconomic factor. The GDPs of these countries represent 91.62% of total GDP (87697519 in 2019). The Table also report the DGP and percentage over world GDP according to World Bank.

Factor correlations

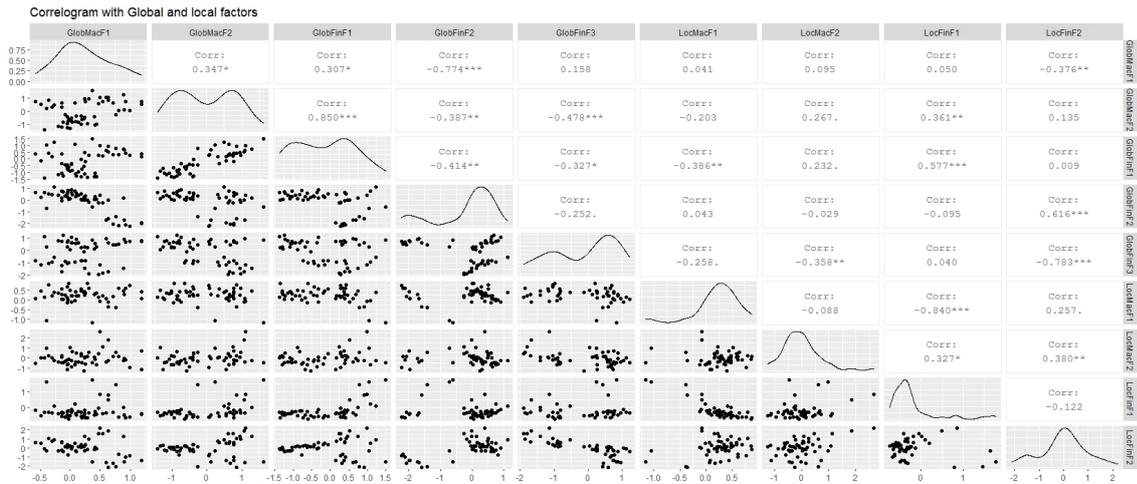


Figure 7: Pairwise correlations between US factors

The Figure plots the pairwise scatter plots between the estimated factors when the factors are estimated separately for each set of variables. To avoid the effect on the estimated correlations of influential points, we have deleted the factors estimated in 2008Q4, 2009Q1 and 2020Q1. We can observe that there are two important correlations to be considered when building the multi-level factor model structure. First, there is a strong positive correlation between the second global macroeconomic factor and the first global financial factor while the first global macroeconomic factor and the second global financial factors are negatively correlated. On the other hand, global and local factors are only correlated when we look at the financial factors with the second local financial factor having a mild positive correlation with the first global and a stronger negative correlation with the third global financial factor.