Fire sales, indirect contagion and systemic stress testing

Rama Cont and Eric Schaanning

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Bank stress tests

- Bank stress tests have become an essentiel component of bank supervision.
- Stress tests assume 'passive' behavior by banks.
  BCBS 2015: "Stress tests conducted by bank supervisors still lack a genuine macro-prudential component" .. "*endogenous* reactions to initial stress.. loss amplification mechanisms and *feedback effects*" are missing.
- Financial institutions subject to portfolio constraints (capital, liquidity, leverage constraints) unwind positions when faced with large losses
  -empirical evidence of deleveraging in stress scenarios (Shleifer 2010, Coval & Stafford 2007, Ellul et al 2011).
  -evidence from banks 'living wills': (Credit Suisse, 2015): *"If we are unable to raise needed funds in the capital markets (...), we may need to liquidate unencumbered assets to meet our liabilities [..] at depressed prices."*

Channels of loss amplification in the financial system

- R Cont and E Schaanning (2016). Fire sales, indirect contagion and systemic stress-testing, `http://ssrn.com/abstract=2541114`
- R Cont and E Schaanning (2019) Monitoring indirect contagion, Journal of Banking and Finance, 104, 85-102.
- R Cont, L Wagalath (2013) Running for the Exit: Distressed Selling and Endogenous Correlation in Financial Markets, **Mathematical Finance**, Vol 23, Issue 4, p. 718-741.
- R Cont, L Wagalath (2016) Fire sale forensics: measuring endogenous risk. **Mathematical Finance**, Volume 26, Issue 4, 835-866.

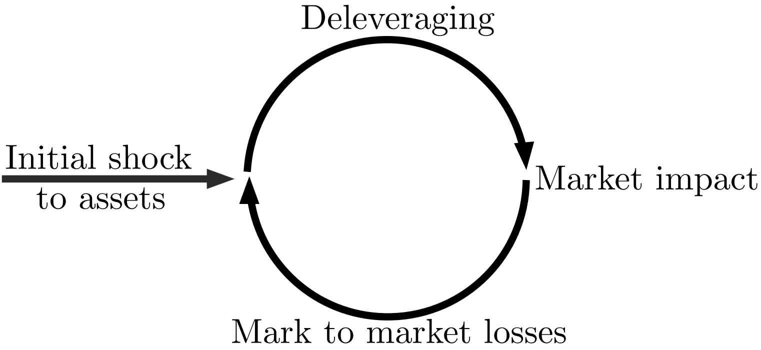
Channels of loss amplification in the financial system

1. Counterparty Risk: balance sheet contagion through asset devaluation = contagion via interbank exposure network
2. Funding channel: balance sheet contagion through withdrawal of funding (bank runs by depositors, institutional bank runs by lenders) = contagion via interbank lending network
3. **Feedback effects from deleveraging**: loss contagion through mark-to-market losses in common asset holdings

Research on financial networks and their use in macroprudential regulation has focused on direct contagion mechanisms (1+2). Regulatory measures have focused on 1 (large exposure limits, central clearing, CVA, ring-fencing) or 2 (LCR, NSFR).

Feedback effects from portfolio deleveraging

(Shleifer 2010, Coval & Stafford 2007, Ellul et al 2011, Kyle & Xiong 2005, Cont & Wagalath 2013,Greenwood et al 2013, Eisenbach & Duarte 2018)



Systemic stress testing with endogenous risk

We build on previous theoretical work on fire sales (Shleifer 2010, Coval & Stafford 2007, Ellul et al 2011, Kyle & Xiong 2005, Cont & Wagalath 2013,..) and recent empirical studies (Greenwood et al 2013, Eisenbach -Duarte 2014) to construct an **operational** framework for quantifying fire sales spillovers and incorporating it in a system-wide stress test for financial institutions.

Ingredients:

1. Network
2. Constraints
3. Reactions
4. Feedback mechanism
5. Contagion

Loss amplification from portfolio deleveraging

(Cont & Schaanning, 2016)

1. Portfolio holdings of financial institutions by asset class: N institutions, K *illiquid* asset classes, M *marketable* asset classes $\rightarrow N \times (M + K)$ portfolio matrix (network)

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4. **Market impact**: market prices react to portfolio rebalancing
5. Mark-to-market accounting: transmits market impact to all institutions $\rightarrow$ may lead to feedback if market losses large

Balance sheets: illiquid vs marketable assets

Illiquid assets
Residential mortgage exposures
Commercial real estate exposure
Retail exposures: Revolving credits, SME, Other
Indirect sovereign exposures in the trading book
Defaulted exposures
Residual exposures

Marketable assets
Corporate bonds
Sovereign debt
Derivatives
Institutional client exposures: interbank, CCPs,...

Table: Stylized representation of asset classes in bank balance sheets. (Data: European Banking Authority 2011)

- Illiquid holdings of institution i: $\Theta^i := \sum_{\kappa=1}^{K} \Theta^{i\kappa}$. Cannot be traded, are held to maturity. Ex: loans.
- Marketable securities: $\Pi^i := \sum_{\mu=1}^{M} \Pi^{i\mu}$. Ex: stocks, bonds.
- Equity (Tier 1 capital): C^i
- Portfolioss are subject to various **one-sided** constraints: leverage ratio, capital ratio, liquidity ratio.
- Leverage ratio of i:

$$\lambda^i = \frac{Assets(i)}{C^i} = \frac{\Theta^i + \Pi^i}{C^i} \leq \lambda_{\max}$$

- Capital ratio of i:

$$\lambda^i = \frac{RWA(i)}{C^i} = \frac{\sum w_\kappa \Theta^{i,\kappa} + \sum_\mu \Pi^{i,\mu} w_\mu}{C^i} \leq R_{\max}$$

  Basel 3 rules: $\lambda_{\max} = 33$, $R_{\max} = 12.5 = 1/0.08$
- Banks maintain a capital/liquidity buffer (slightly) above the regulatory requirements $\rightarrow$ target leverage ratio $\lambda_b^i < \lambda_{\max}$, target capital ratio $R^i < R_{\max}$.

Deleveraging

- Observation: when portfolio constraints are breached following a loss in asset values, financial institutions **deleverage** their portfolio by selling some assets in order to comply with the portfolio constraint.

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Deleveraging assumption: if following a loss L^i in asset values, leverage of bank i exceeds constraint,

$$\lambda^i = \frac{\Theta^i + \Pi^i - L^i}{C^i - L^i} > \lambda_{\max}$$

bank deleverages by selling a proportion $\Gamma^i \in [0,1]$ of assets in order to restore a leverage ratio $\lambda_b^i \leq \lambda_{\max}$:

$$\frac{(1-\Gamma^i)\Pi^i + \Theta^i - L^i}{C^i - L^i} = \lambda_b^i \leq \lambda_{\max} \quad \Rightarrow \Gamma^i = \frac{C^i(\lambda^i - \lambda_b^i)}{\Pi^i} \mathbf{1}_{\lambda^i > \lambda_{\max}}$$

Develeraging in response to a loss

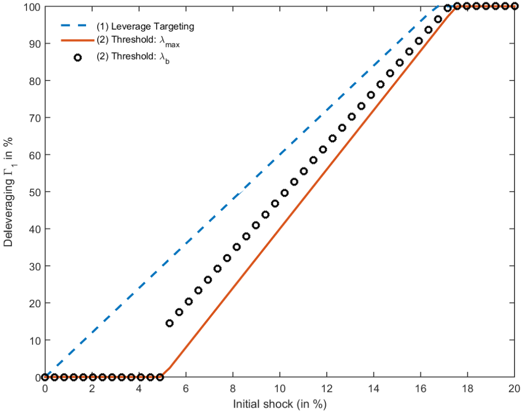

Figure: Percentage of marketable asset deleveraged in response to a shock to assets (circles) for a leverage constraint of 20. Leverage targeting (dotted blue) would lead to a linear response.

Market impact and Feedback effects

Total liquidation in asset μ at k-th round: $q^{\mu} = \sum_{j=1}^{N} \Pi_k^{j,\mu} \Gamma_{k+1}^{j}$

$$\text{Market impact}: \quad \frac{\Delta S^{\mu}}{S^{\mu}} = -\Psi_{\mu}(q^{\mu}),$$

Impact/ inverse demand function: $\Psi_{\mu} > 0, \Psi'_{\mu} > 0, \Psi_{\mu}(0) = 0.$

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Price move at k-th iteration of fire sales:

$$S_{k+1}^\mu = S_k^\mu \left(1 - \Psi_\mu \left(\sum_{j=1}^{N} \Pi_k^{j,\mu} \Gamma_{k+1}^j\right)\right),$$

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$$\Pi_{k+1}^{i,\mu} = \underbrace{\left(1 - \Gamma_{k+1}^i\right)}_{\text{Non-liquidated assets}} \overbrace{\Pi_k^{i,\mu}}^{\text{Previous value}} \underbrace{\left(1 - \Psi_{\mu} \left(\sum_{j=1}^{N} \Pi_k^{j,\mu} \Gamma_{k+1}^j \right) \right)}_{\text{Price impact on remaining holdings}}$$

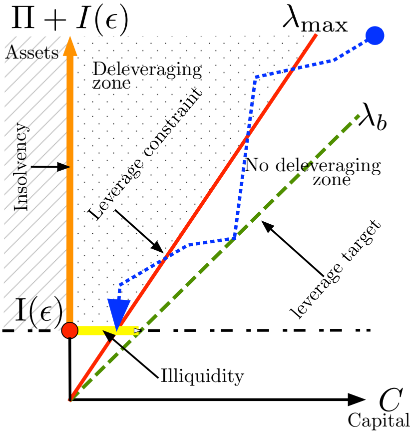


Figure: Portfolio constraints define a set of admissible portfolios. A large loss may take the portfolio outside this set, in which case banks deleverage in order to revert back to this set.

Portfolio overlaps as drivers of loss contagion

When market impact is linear(ized) $\Psi_\mu(x) = x/D_\mu$ (where $D_\mu =$ market depth) the mark-to-market loss of i resulting from fire sales is

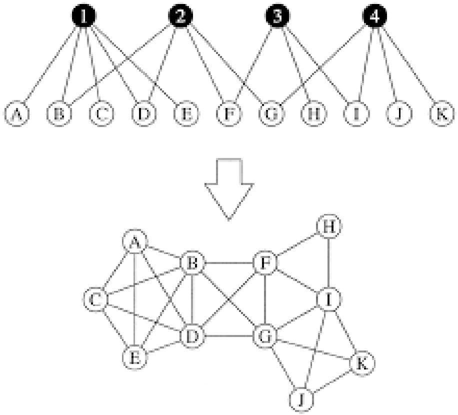
$$L^i = \sum_{j=1}^{N} \underbrace{\sum_{\mu=1}^{M} \frac{\Pi^{i\mu}\Pi^{j\mu}}{D_\mu}}_{\Omega_{ij}} \Gamma^j = \sum_{j=1}^{N} \Omega_{ij}\Gamma^j,$$

where Ω_{ij} is the **liquidity weighted overlap** between portfolios i and j (Cont & Wagalath 2013):

$$\Omega_{ij} = \sum_{\mu=1}^{M} \frac{\Pi^{i\mu}\Pi^{j\mu}}{D_\mu} \qquad D_\mu = \text{market depth for asset } \mu$$

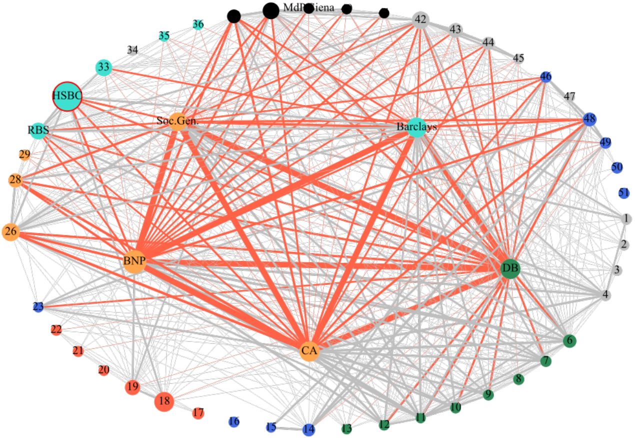
$\Omega_{ij} =$ exposure of marketable assets of i to 1% deleveraging by j.
$\Rightarrow$ loss contagion = contagion process on network defined by $[\Omega_{ij}]$

Bipartite network of asset holdings

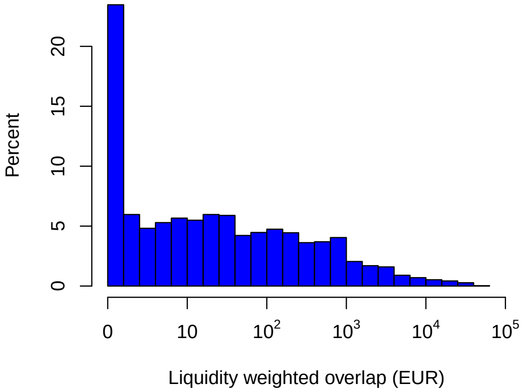


Indirect exposures across institutions through common asset holdings

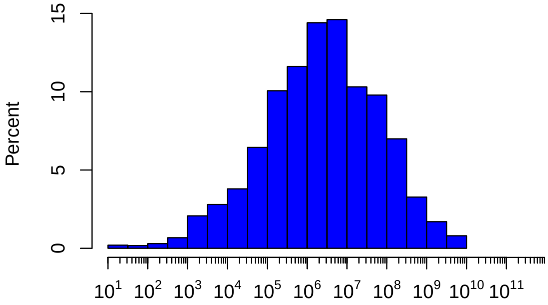
The EU indirect contagion network (2016)



Portfolio overlaps across EU banks (EBA 2011)

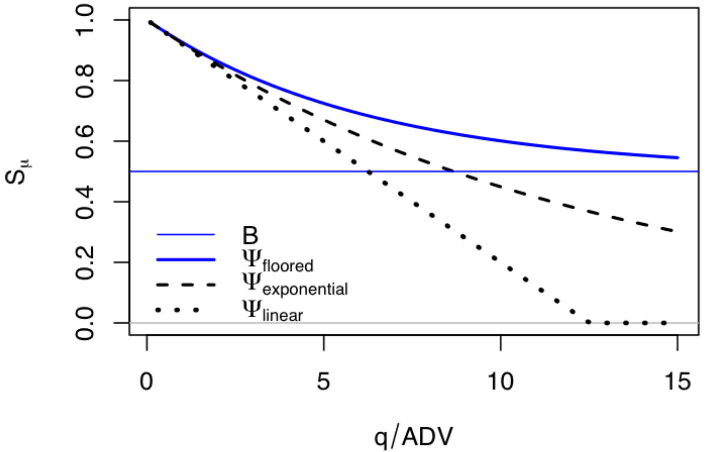


2nd round overlaps across EU banks (EBA 2011)



Distribition of elements of Ω^2 representing 2nd round spillover effects.

Market impact function



Market impact function and market depth

The impact of a total distressed liquidation volume q is modelled by a *level-dependent market impact function*

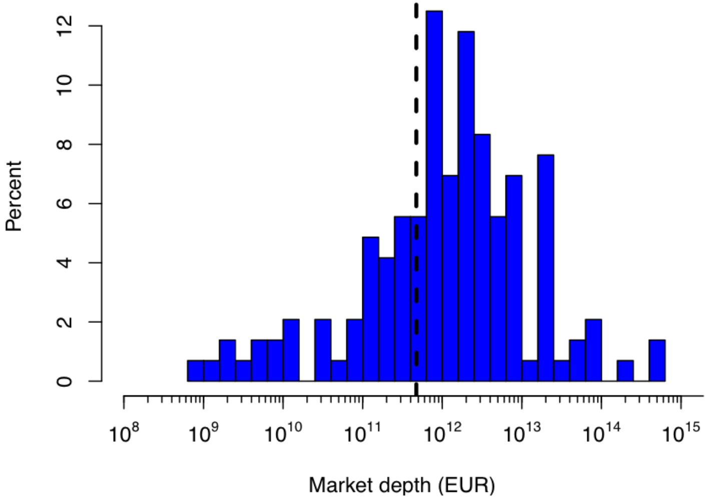
$$\Psi_\mu(q, S) = \left(1 - \frac{B_\mu}{S}\right)\left(1 - \exp\left(-\frac{q}{D_\mu}\right)\right),$$

where

$$D_\mu = c\frac{ADV_\mu}{\sigma_\mu}\sqrt{\tau},$$

- $S \geq B_\mu$ where B_μ is the price-floor
- ADV: average daily volume, σ_μ: daily volatility of asset
- $c \approx 0.25$, a coefficient to make Ψ_μ consistent with empirical estimates of the linear impact model for small volumes q.
- τ is the liquidation horizon

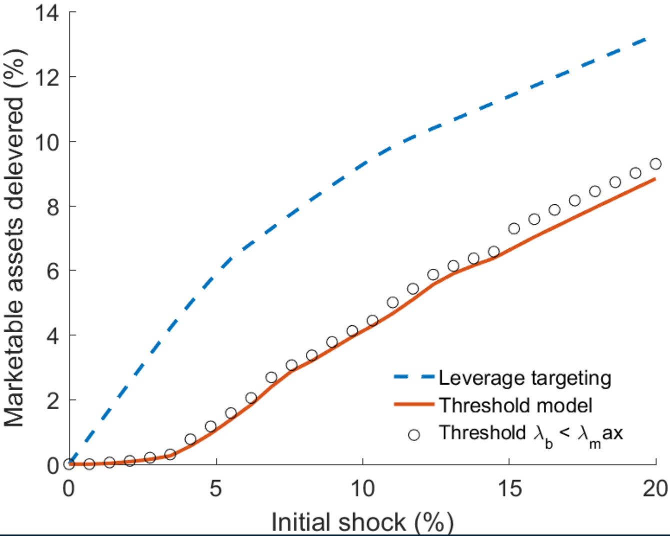
Estimated market depth



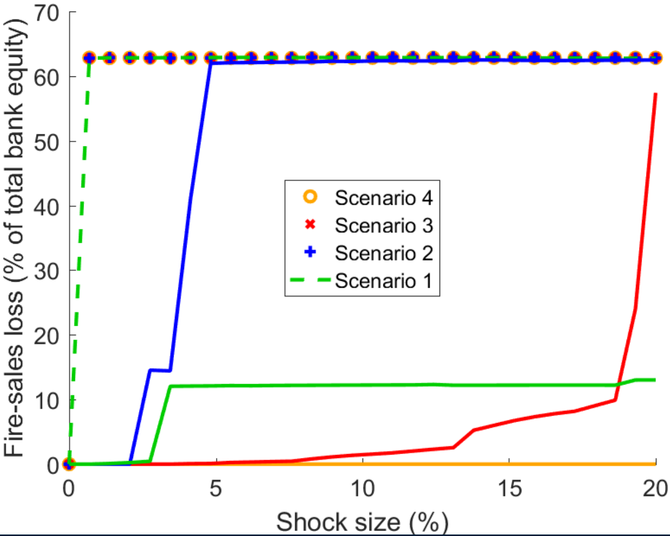
Stress scenarios

- A stress scenario is defined by a vector $\epsilon \in [0,1]^K$ whose components ϵ_κ are the percentage shocks to asset class κ.
- Initial/Direct loss of portfolio i: $L_i^0(\epsilon) = \epsilon.\Pi^i = \sum_\kappa \Pi^{i\kappa}\epsilon_\kappa$
- We consider the EBA stress scenarios used in the actual EU 2016 stress test and modulate the shock sizes ϵ_κ from 0% to 20%.
- Examples of stress scenarios:
    1. Spanish residential and commercial real estate losses
    2. Northern Europe residential losses
    3. Southern Europe commercial real estate losses
    4. Eastern Europe commercial real estate losses

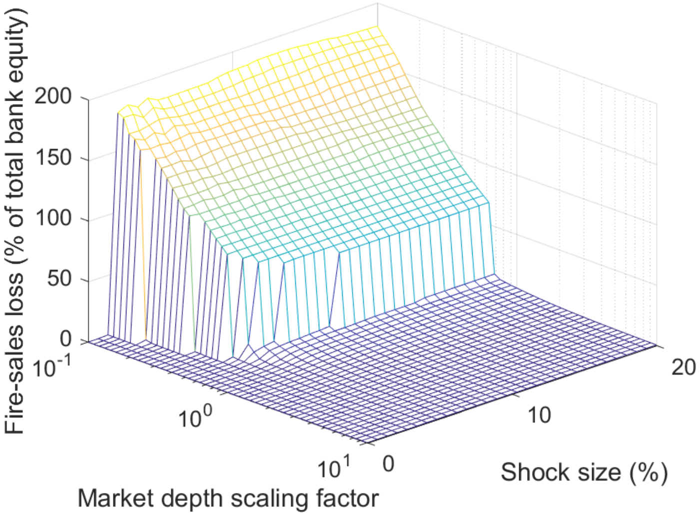
Fire sales losses



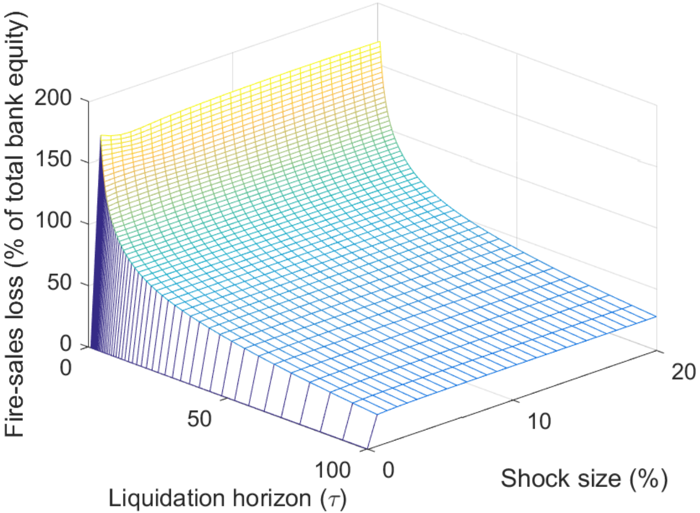
Indirect losses: existence of a critical shock size



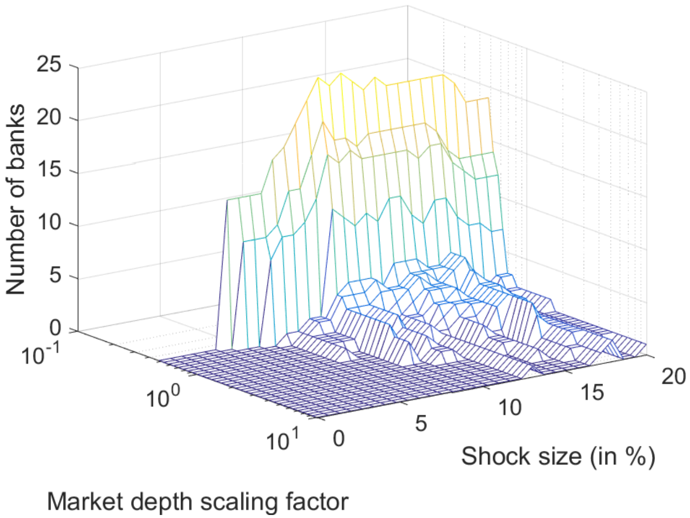
Fire sales losses and market depth



Impact of liquidation horizon



Endogenous losses modify stress test outcomes



Failures due to illiquidity and insolvency

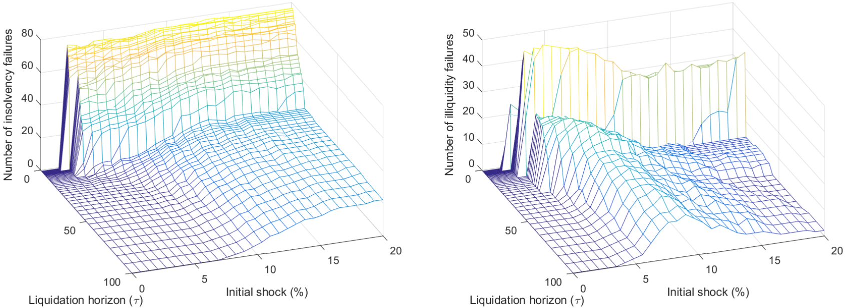


Figure: The model allows to distinguish between failures due to insolvency (negative equity - left) and failures due to illiquidity (zero liquid assets - right).

Indirect exposures

Consider two institutions (A) and (B).

- A and B hold a common financial asset (say, gov bonds). A holds an illiquid asset ('subprime') that B does not hold. Notional exposure of B to 'subprime' is zero.
- However, in the event of a large loss in 'subprime' assets, A may be forced to sell some of its bonds, pushing down their market price, resulting in a market loss for the B.
- So: B experiences a loss following a large shock to 'subprime' assets: B has an (indirect) exposure to an asset it does not hold!
- Magnitude of this indirect exposure is directly linked to the overlap between B and institutions holding this asset.
- Institutions with large holdings across multiple asset classes increase overlaps across system and become vectors of indirect contagion.

Indirect exposures

In a scenario where a shock ϵ_κ is applied to asset class κ,
Total loss = Direct loss + indirect loss through contagion

$$Loss(i, \epsilon_\kappa) = \underbrace{\epsilon_\kappa \; \Theta^{i,\kappa}}_{\text{Direct Loss}} + \underbrace{\sum_{j=1}^{N} \sum_{\mu=1}^{M} \frac{\Pi^{i\mu} \Pi^{j\mu}}{D_\mu} \Gamma^j(\epsilon)}_{\text{Indirect Loss}}$$

Indirect exposures arise from the 2nd term, which can be > 0 even if $\Theta^{i,\kappa} = 0$ i.e. bank i does not even hold asset class κ.

Indirect exposures

The **effective exposure** of institution i to asset class κ is given by

$$E^{i,\kappa}(\epsilon_\kappa) := \frac{\mathrm{Loss}(i,\epsilon_\kappa)}{\epsilon_\kappa} = \underbrace{\Theta^{i,\kappa}}_{\text{Notional exposure}} + \underbrace{\frac{ILoss(i,\epsilon_\kappa)}{\epsilon_\kappa}}_{\textbf{Indirect exposure}},$$

where $ILoss(i,\epsilon_k)$ is the *total indirect loss* of$i$ in a scenario where a shock ϵ_κ is applied to asset class κ.

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where $ILoss(i,\epsilon_k)$ is the *total indirect loss* of$i$ in a scenario where a shock ϵ_κ is applied to asset class κ. The effective exposure is scenario dependent and accounts for losses that i would suffer in a stress scenario.

$\rightarrow$ it reflects the network-dependent (and actual!) risk of i's portfolio.

Losses arising from indirect exposures

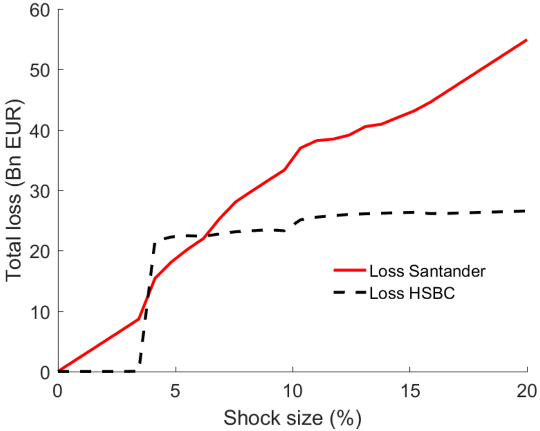


Figure: Losses of HSBC and Banco Santander as a function of losses in the Southern European real estate sector.

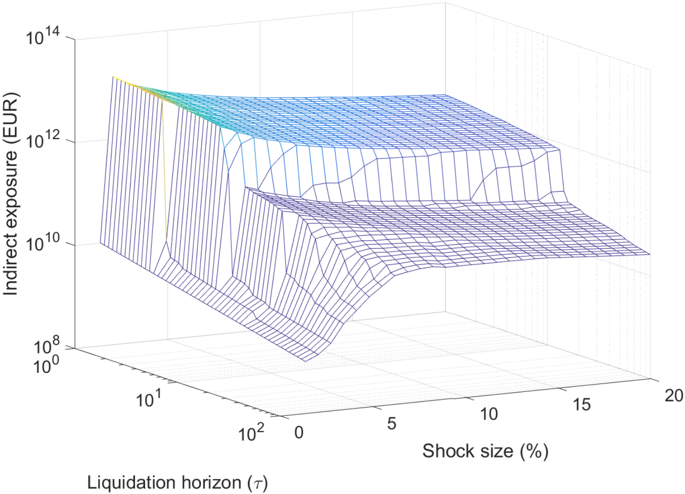


Figure: Indirect exposures of UK banks to Southern European real estate.

Monitoring indirect contagion

Portfolio overlaps as drivers of Indirect contagion

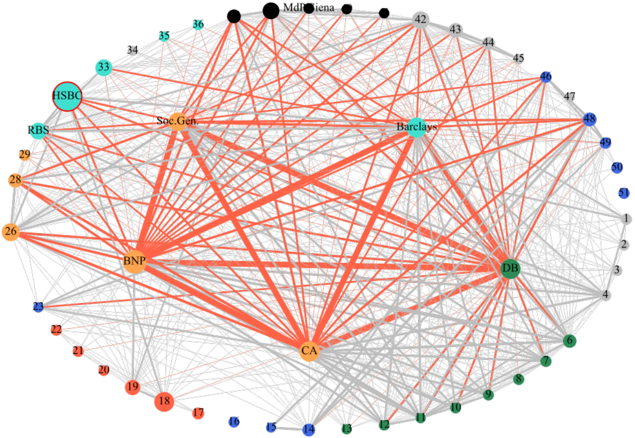
When market impact is linear, the mark-to-market loss of i resulting from fire sales is given by

$$L^i \approx \sum_{j=1}^{N} \underbrace{\sum_{\mu=1}^{M} \frac{\Pi^{i\mu}\Pi^{j\mu}}{D_\mu}}_{\Omega_{ij}} \Gamma^j = \sum_{j=1}^{N} \Omega_{ij}\Gamma^j,$$

where Ω_{ij} is the *liquidity weighted overlap* between portfolios i and j (Cont & Wagalath 2013).

Thus: price mediated contagion can be modeled as a contagion process on a network whose nodes are financial institutions and whose links are weighted with liquidity weighted overlaps.

The EU indirect contagion network (2016)



Principal component analysis of liquidity-weighted overlaps

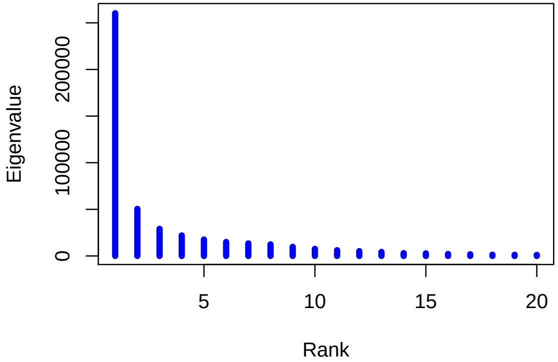


Figure: European banking system: Eigenvalues of matrix of liquidity-weighted overlaps. Source: EBA (public)

Indirect Contagion Index

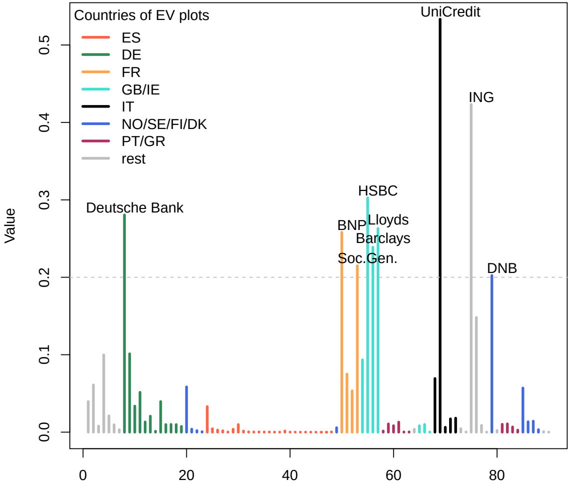
The principal eigenvector $U = (U_i, i = 1...N)$ corresponding to the largest eigenvalue of the matrix of liquidity-weighted overlaps provides a measure of (eignevector) centrality of the node i in the indirect contagion network

Definition (Indirect Contagion Index (ICI))

We define the **Indirect Contagion Index (ICI)** of a financial institution i as its component U_i in the (normalized) principal eigenvector of the matrix of liquidity weighted portfolio overlaps:

$$ICI(i) = U_i$$

Liquidity weighted overlaps: 1st principal component



Indirect Contagion Index as a measure of exposure to fire sales loss

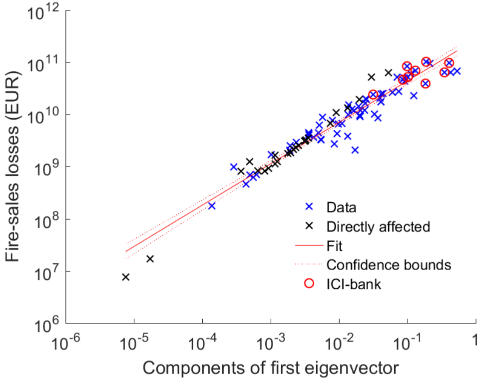
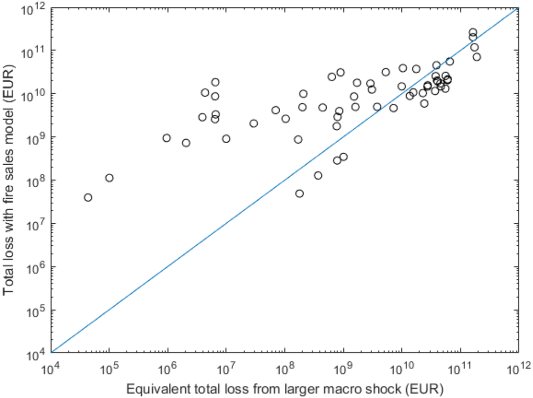


Figure: Regression of $\log(FLoss^i)$ on $\log(ICI_i)$ for a 13% shock at estimated market depth. $R^2 = 0.89$.

Indirect contagion effects cannot be mimicked by scaling up macro shocks

Scaling up the macro shocks can replicate the average bank loss but not the cross-sectional distribution of losses across banks.



Summary

Quantitative model for fire sales spillovers in a network of institutions with common asset holdings subject to *one-sided* portfolio constraints:

- **Tipping point:** Existence of critical macro shock level beyond which fire sales are triggered and significant contagion occurs. In EU banks: threshold large – but not extreme.

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- **Fire sales losses:** Even with optimistic estimates of market depth, fire sales losses can amount to over 20% of system bank equity. This is significant enough to *change the outcome* of stress tests.
- **Heterogeneity of bank losses:** The cross sectional distribution of losses due to fire sales *cannot* be replicated by simply applying a larger initial macro-shock to all assets or banks.

Summary

- **Illiquidity and insolvency:** Our model allows to distinguish between failures due to insolvency and defaults due to illiquidity. Ignoring failures due to illiquidity may lead to a severe underestimation of the extent of contagion.
- **Indirect exposures:** Our model leads to a quantifiable notion of *indirect* exposure to an asset class. EU banks are shown to have significant exposure to housing markets in *other* European countries.
  $\rightarrow$ Calls for a re-thinking of macro-prudential regulation at the national level.
- **Indirect contagion index:** Liquidity-weighted overlaps lead to a bank-level indicator that may be used for monitoring and for quantifying the contribution (and vulnerability) of a financial institution to price-mediated contagion;

Implications for macroprudential supervision and policy

- Incorporating bank reactions greatly alters the outcome of the stress tests
- Capital adequacy should be examined in the light of systemic stress tests incorporating such endogenous effects and contagion mechanisms
- Fire sales and the resulting price-mediated contagion leads to significant **indirect exposures** across sectors and countries. Systemic stress tests allow to evaluate these indirect exposures.
- **Disseminating indirect exposures** can help financial institutions manage and internalize this risk.
- Most failures occurs through illiquidity, not insolvency: suspension of mark-to-market accounting for illiquid assets does not necessarily help this.

References

- R Cont and E Schaanning (2016) Fire sales, indirect contagion and systemic stress-testing, `http://ssrn.com/abstract=2541114`
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