Inflation in a Changing Environment
Price Setting: Synchronization

Fernando Alvarez

University of Chicago

September 2019

"Price synchronization and cost pass-through in multiproduct firms: Evidence from Danish producer prices"

By L. Dedola, M. Kristoffersen and Zullig

- Very rich data set firms: prices, quantities, cost and other variables
- This paper has several interesting findings, but more to come.
- Review some, provide some model for interpretation, comments.

- Set-up simple multiproduct model
- Kurtosis & Area under IRF of small monetary shock.
- Discuss common vs idiosyncratic shocks: implications for Kurtosis
- Discuss effect of (small) trend inflation and implications
- Discuss measurement error and Kurtosis.
- Comment on differential pass-through.

Firm's problem: approx. to CES demand + CRTS

$$V(p) = \min_{\{\tau_j,\ \Delta p(\tau_j)\}_{j=1}^{\infty}} \mathbb{E}\left[\sum_{j=1}^{\infty} e^{-r\tau_j}\ \psi + \int_0^{\infty} e^{-rt} B \left(\sum_{i=1}^{n} p_i^2(t) \right) dt \,\middle|\, p(0) = p \right]$$

where

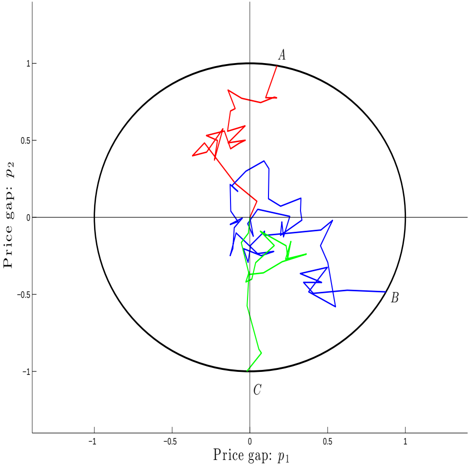
$$p_i(t) = \sigma \mathcal{W}_i(t) + \sum_{j:\tau_j < t} \Delta p_i(\tau_j) \quad \text{for all } t \geq 0 \text{ and } i = 1, 2, ..., n\ ,$$

- $p_i(t)$ percentage deviation of price i from its optimal frictionless value
- stopping times τ_j and adjustments $\Delta p_i(\tau_j)$ all $i = 1, .., n$ and $j = 1, 2, ...$
- $\mathrm{d}p_i = \sigma\ \mathrm{d}\mathcal{W}_i$: n Independent Brownian Motions (prod. shocks).
- pay fixed cost ψ (fraction of profits) and adjust prices of all products Δp.

Key idea: summarize state by scalar: $y \equiv ||p||^2$

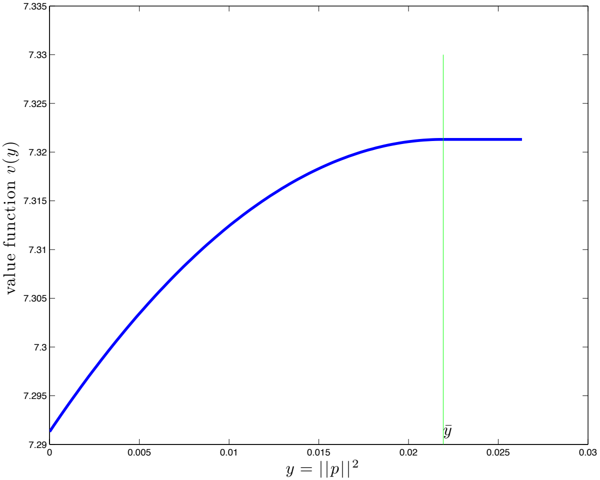
$y \equiv ||p||^2$ square of a **Bessel** process: : $\mathrm{d}y = n\,\sigma^2\,\mathrm{dt} + 2\,\sigma\sqrt{y}\,\mathrm{d}\mathcal{W}$

Inaction region = sphere: $\mathcal{I} = \{p : ||p||^2 \leq \bar{y}\}$.



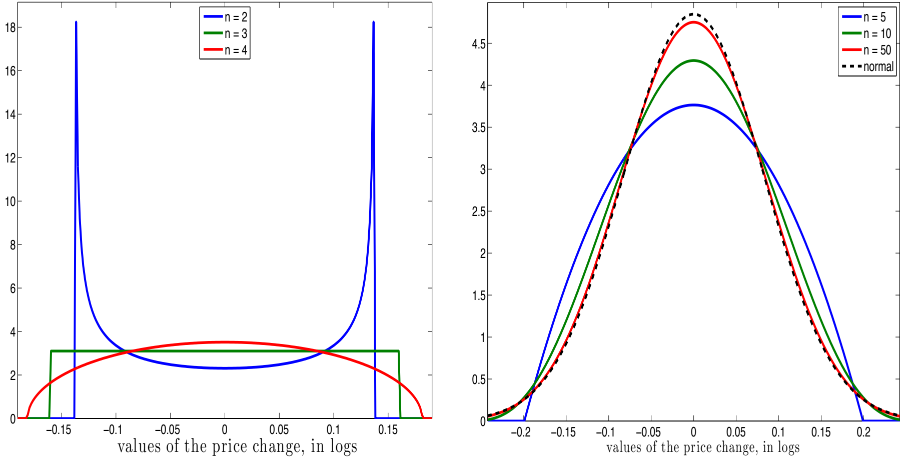
$v(y) = v\left(||p||^2\right) = V(p_1, ..., p_n)$

Value function $v(y) = v(||p||^2) = V(p_1, ..., p_n)$



Each y corresponds to a square radius of vector p

Density $w(\cdot)$ of the price changes as n varies



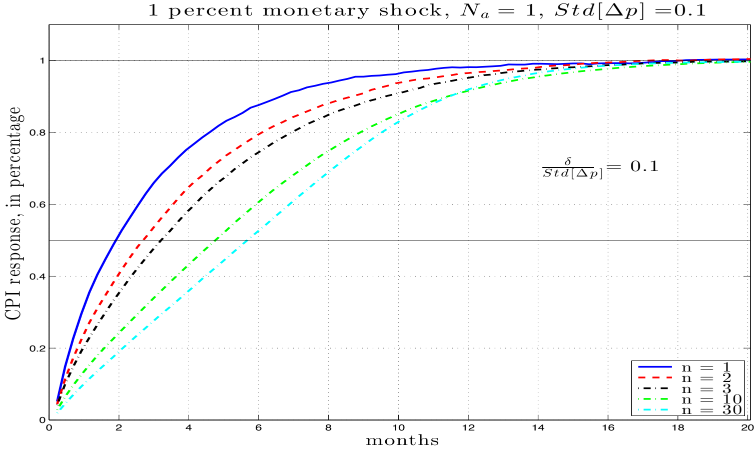
Fixing $\bar{y}$ density w depends only on n.
As n increases, change on prices of each product are *"more independent"*.
Kurtosis of Price change $= \frac{3n}{2+n}$

Some price-setting statistics that depend ONLY on n

	Number of products n					
	1	2	4	6	10	50
Statistics	Model statistics					
$Std(\lvert \Delta p_i \rvert) \;/\; E(\lvert \Delta p_i \rvert)$	0	0.48	0.62	0.65	0.70	0.75
Kurtosis(Δp_i)	1.0	1.5	2.0	2.25	2.5	2.88
Fraction: $\lvert \Delta p_i \rvert < \frac{1}{2} E(\lvert \Delta p_i \rvert)$	0	0.21	0.27	0.28	0.30	0.31
Fraction: $\lvert \Delta p_i \rvert < \frac{1}{4} E(\lvert \Delta p_i \rvert)$	0	0.10	0.13	0.14	0.15	0.16

back

$\mathcal{P}_n(\delta, t)$: IRF (log) CPI to shock $\delta = 1\%$



- IRF for (log) output $Y(t) = \delta - \mathcal{P}_n(\delta, t)$
- For small shock, it is Only function of N_a and n

Effect of Monetary Shocks

- Let $\mathcal{M}(\delta)$ be the area under the impulse response function (IRF) of output to a monetary shock of size δ.
- Monetary shock is a once and form all increase in money (or costs) in δ.
- Let $Kurt\,(\Delta p)$ be the kurtosis of price change in steady state.
- Let N_a be the kurtosis of price change in steady state.
- Then, for a small monetary shock δ:

$$\mathcal{M}(\delta) = \frac{Kurt\,(\Delta p)}{6\,N_a}\,\delta$$

- Entire IRF -not just area- characterized by eigenfunctions-eigenvalues.

Sensitivity to trend inflation μ

- Static “target" prices have drift μ, all price gaps drift down
- Optimal decision rule are different (no closed form)
  - Prices are *not* reset to static target at adjustment.
  - Inaction set $\mathcal{I}$ is *not* a hyper-sphere.
- Inflation has only second order effect around $\mu = 0$ on
  - *frequency* of price changes N_a,
  - all centered *even moments* of marginal price changes (e.g. kurtosis).
- Inflation has first order effect on difference in *Size* and Frequency of Price Increases *minus* Decreases
  - For $n = 1$ can show that 90% of adjustment to inflation μ is difference in frequency increases vs decreases, 10% in size. (QJE)

Modeling drift and correlation

Let each price gap follow (inflation μ, correlation $\frac{\bar{\sigma}^2}{\bar{\sigma}^2+\sigma^2}$)

$$\mathrm{d}p_i = -\mu\,\mathrm{d}t + \bar{\sigma}\,\mathrm{d}\bar{\mathcal{W}} + \sigma \mathrm{d}\mathcal{W}_i \text{ for all } i = 1, ..., n\,.$$

where $\bar{\mathcal{W}}, \mathcal{W}_i$ are independent standard BMs. Define:

$$y = \sum_{i=1}^{n} (\,p_i\,)^2 \quad \text{and} \quad z = \sum_{i=1}^{n} p_i$$

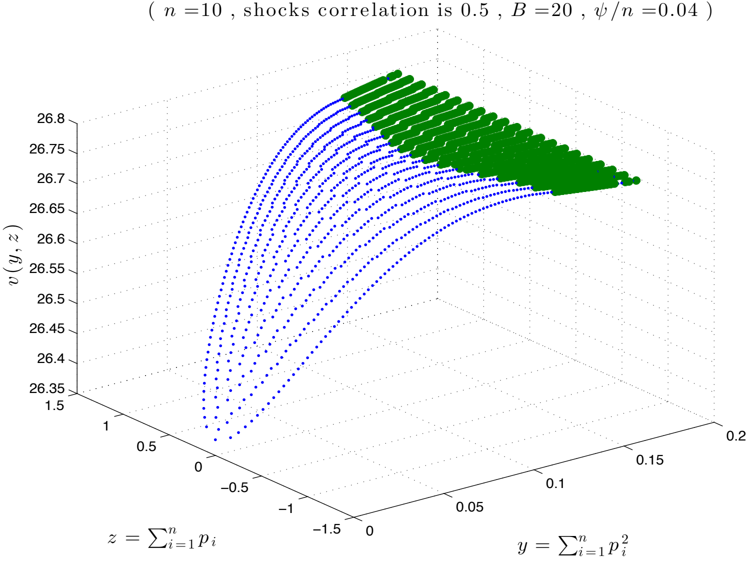
Using Ito's Lemma define the diffusions

$$\begin{aligned} \mathrm{d}y &= \left[n\sigma^2 + n\bar{\sigma}^2 - 2\mu z\right]\mathrm{d}t + 2\sigma\sqrt{y}\;\mathrm{d}\mathcal{W}^a + 2\bar{\sigma}z\;\mathrm{d}\mathcal{W}^c \\ \mathrm{d}z &= -n\mu\,\mathrm{d}t + n\,\bar{\sigma}\;\mathrm{d}\mathcal{W}^c + \sqrt{n}\sigma\left(\frac{z}{\sqrt{ny}}\mathrm{d}\mathcal{W}^a + \sqrt{1-\left(\frac{z}{\sqrt{ny}}\right)^2}\,\mathrm{d}\mathcal{W}^b\right) \end{aligned}$$

where $(\mathcal{W}^a, \mathcal{W}^b, \mathcal{W}^c)$ are three standard independent BM's.

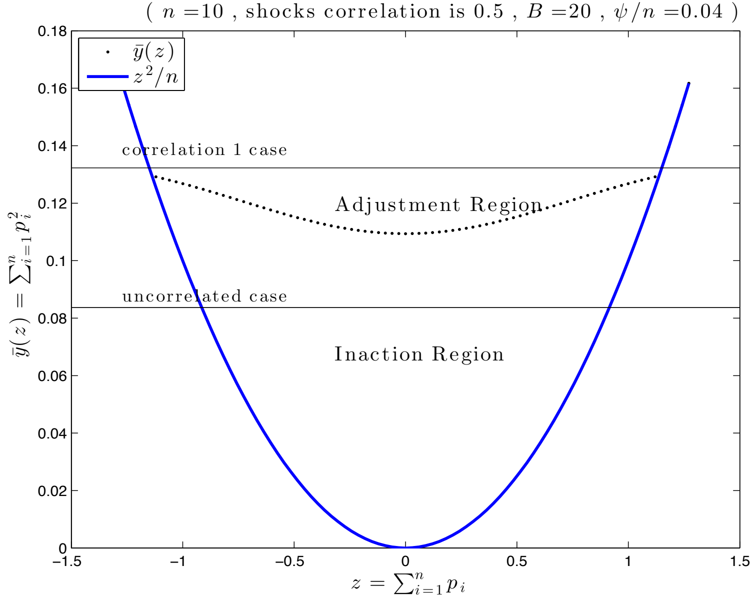
- Only two dimensions for decision rule and IRF!

Value function $v(y, z)$ and decision rules: no drift

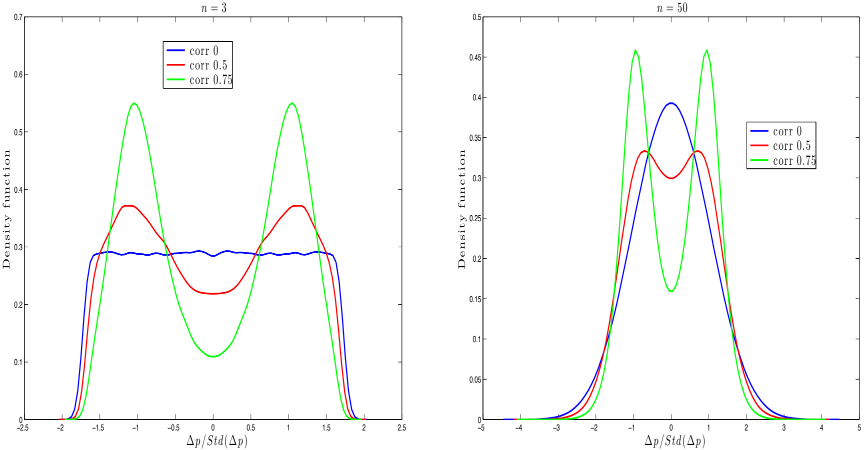


Similar to lower n, i.e. it lower Kurtosis

Feasible Set and Inaction Set (no drift)

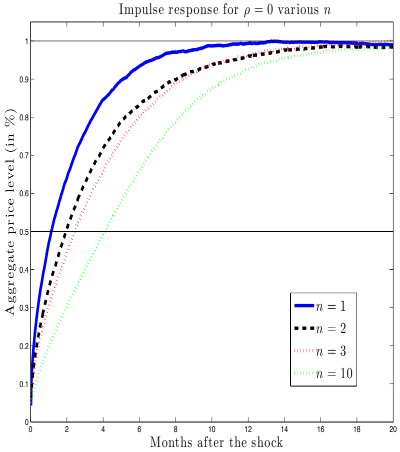


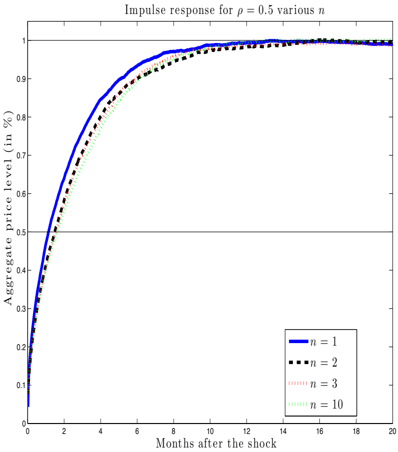
Effect of correlation on distribution $w(\Delta p)$



- Summary: correlation makes is closer to one good.
  It lower Kurtosis, and hence Output IRF.

Impulse response to a monetary shock As expected more flexible, smaller output IRF





Kurtosis in $w(\Delta p)$ and Calvoness

Modify model

- introducing random (free) adjustment opportunities
- Adjustments: **either** if opportunity arrives **or** y reaches $\bar{y} \implies$

  - price changes **mixture** of distributions with $Var(\Delta p) = \frac{y}{n}$ for all $y \leq \bar{y}$.

Main Results

- Introduce even more small price changes. Limit case is *Laplace*
- Optimal policy $\bar{y}$ with (r, λ) same as with $(r + \lambda, 0)$

  Intuition: effective discount rate of cost $r + \lambda$
- While decision rules are of the same form, frequency of price changes, invariant distribution of price gaps, and distribution of price changes all change.
- Hazard rate $h(t)$: just adds constant λ at all elapsed times t.

Multi-product version of Calvo$^+$

Fix $\lambda > 0, \sigma > 0$ and $n \geq 1$.

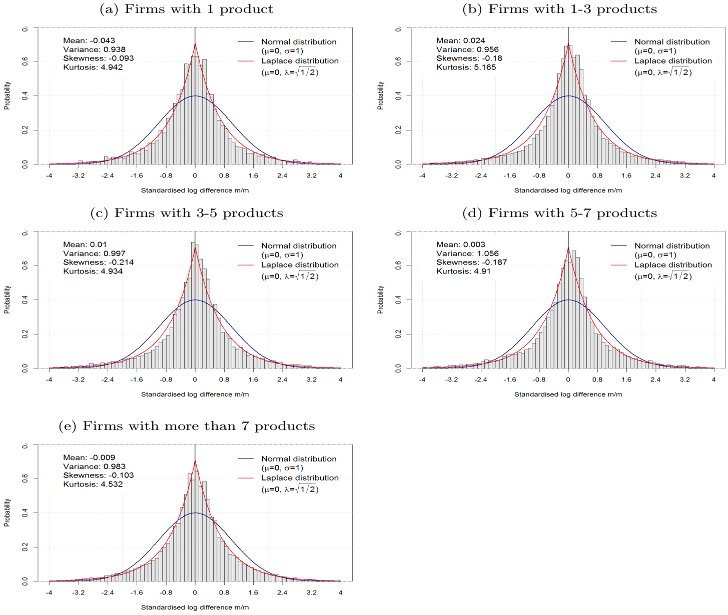
(i) $Kurt(\Delta p_i)$ depends on two parameters: n and $\frac{\lambda}{N_a}$

(ii) Let $\psi / B \to \infty$ so that $\bar{y} \to \infty$. Then $N_a \to \lambda$ and $Kurt(\Delta p_i) \to 6$ (Laplace)

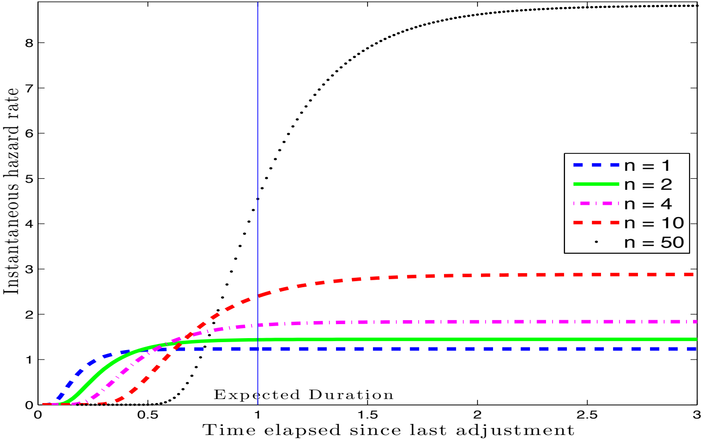
Table: Kurtosis of Price changes: $\mathbb{E}(\Delta p_i)^4 / (\mathbb{E}(\Delta p_i)^2)^2$

% of free adjustments:	number of products n					
λ / N_a	1	2	4	6	10	50
0%	1.0	1.5	2.0	2.2	2.5	2.9
10%	1.1	1.6	2.1	2.4	2.6	3.0
50%	1.6	2.2	2.7	3.0	3.2	3.6
95%	3.5	4.3	4.7	4.9	5.0	5.2
100%	6.0	6.0	6.0	6.0	6.0	6.0

Figure 7: Histograms of standardized price changes



Hazard rate of price changes as *n* varies



Giving expected time between adjustment, hazard depends only on *n*.

As *n* increases, change on prices of each product are *"more independent"*.

go back

Two comments on Measurement Error

- Effect on Kurtosis of price changes
- Effect on differential pass-through coefficient

Kurtosis and Measurement Error

- Distribution of price changes is leptokurtic, more than normal but less than Laplace
- This results differs from US data on PPI.
- I believe because data in this paper has been properly demeaned (heterogeneity,i.e mixing increases kurtosis)
- Kurtosis similar to the one CPI data in France, once corrected by measurement error

  (AER w/LeBihan and Lippi, corrected b comparing CPI w/scanner data)
- Could Kurtosis be even smaller?

Difference in pass-through coefficients

- Passthrough coefficient for energy cost changes is much larger than for imported good cost changes.
- Cost changes are the product of share for the firm times price of the imported good.
- True shares may depend on the good, not just the firm.
- This gives classical measurement error on RHS variable, and hence attenuation bias.
- Heterogeneity on shares at the level of the good can be larger for imported inputs.

"Multi-Product Pricing: Theory and Evidence From Large Retailers in Israel"

By M. Bonomo, C. Carvalho, O. Kryvtsov, S. Ribon and R. Rigato

- Thorough analysis of synchronization of price changes for a large retailer
- New price setting model with infinitely many products and two costs.
- Very nice characterization of output IRF's.
- Review some, provide some model for interpretation, comments.

- Discuss model, by first presenting finitely many products version of it.
- Cost of adjusting any product, and extra cost for each product.
- Present an alternative style estimation of τ^*.

Finitely many product version

- Let $y_i(t) = p_i(t)^2$ the square of each uncontrolled price gap follow
- each $p_i(t)$ follow standard independent BMs w/volatility σ

$$dy_i(t) = \sigma^2 \, dt + 2 \, \sigma \, \sqrt{y_i(t)} \, d\mathcal{W}_i(t) \;\; \text{for all } i = 1, ..., n$$

- firms that adjust $1 \leq m \leq n$ product's prices pays $\psi + m \, \nu$
- ψ independent of the number of product
- ν per product.
- We will write the value function with vector of square price gaps $(y_1, ..., y_n)$ as arguments

Value function $v(y_1, y_2, ..., y_n)$

$$rv\left(y_1, y_2, ..., y_n\right) \leq B\sum_{i=1}^{n} y_i + \sigma^2 \sum_{i=1}^{n} \frac{\partial v\left(y_1, ..., y_n\right)}{\partial y_i} + 2\sigma^2 \sum_{i=1}^{n} \frac{\partial^2 v\left(y_1, ..., y_n\right)}{\partial y_i^2}\, y_i$$

with equality if $(y_1, y_2, ..., y_n)$ is in the inaction region and

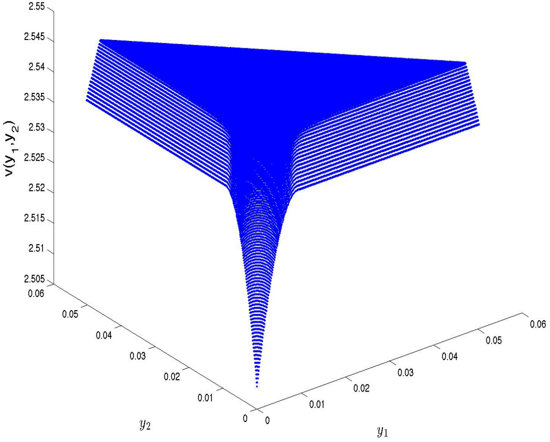
$$v\left(y_1, y_2, ..., y_n\right) \leq \min_{I_j \in \{0,1\}, j=1,\dots,n} \left\{ \psi + \sum_{j=1}^{n} \nu(1 - I_i) + v\left(I_1 y_1, I_2 y_2, ..., I_n y_n\right) \right\}$$

with equality if $(y_1, y_2, ..., y_n)$ is the control region

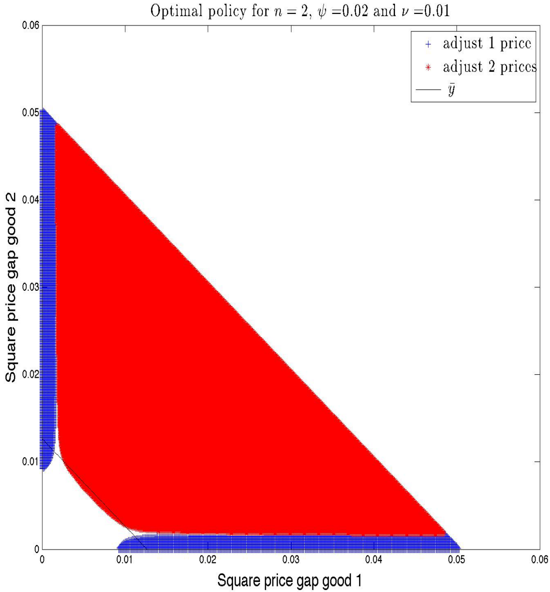
$I_j = 1$ is an indicator that the firm keeps the j-th price in the inaction region.

<u>Example</u> $n = 2$: Inaction, change one price, or change two prices.

Value function for $n = 2$, ψ =0.02 and ν =0.01



Parameters: $r = 0.05$, $B = 20$, $\sigma = 0.1$, $\nu = 0.01$, and $\psi = 0.02$.



Parameters: $r = 0.05$, $B = 20$, $\sigma = 0.1$, $\nu = 0.01$, and $\psi = 0.02$. Solid line $y_1 + y_2 = \bar{y}$ with cost $\psi' = \psi + 2\nu$ and $\nu' = 0$. Decision rules only displayed for points with $y_1 + y_2 < 4\,\bar{y}$.

Table: Statistics for price changes as function of cost ν and ψ

statistics $\backslash$ ψ $2\,\nu$	0.04 0.00	0.035 0.005	0.03 0.01	0.02 0.02	0.01 0.03	0.00 0.04
Fraction simultaneous price changes	1.00	0.83	0.75	0.56	0.34	0.00
Kurtosis(Δp_i)	1.50	1.30	1.21	1.11	1.04	1.00
Fraction: $\lvert\Delta p_i\rvert < \frac{1}{2}E(\lvert\Delta p_i\rvert)$	0.21	0.11	0.06	0.00	0.00	0.00
Fraction: $\lvert\Delta p_i\rvert < \frac{1}{4}E(\lvert\Delta p_i\rvert)$	0.11	0.00	0.00	0.00	0.00	0.00

- As “marginal" fixed cost ν increases relative to "fixed" fixed ψ:
- Lower synchronization and Lower Kurtosis of price changes

Partial Synchronization, Kurtosis and output IRF

- Model in the paper has infinitely many products in the firm
- In the model all shocks are independent within the firm.
- Firm change prices every τ^* periods, to save in "fixed" fixed cost ψ.
- Synchronize price changes, as in $n \to \infty$ in Alvarez and Lippi.
- But marginal fixed cost $\nu > 0$ implies that firm does NOT change all products, just the ones with large price gaps.
- On the other hand, it reduces Kurtosis and reduces output IRF!
- Conjecture: Kurtosis lies between 1 and 3 (in benchmark in paper $\approx$ 1)
- Please compute Kurtosis of price changes & cumulative output's IRF!

Estimate τ^*, σ^2 & area outside inaction

- Estimate (frequency of price changes) N_a as usual.
- For σ^2 can use lemma in AL (AER): $N_a \, Var(\Delta p) = \sigma^2$
- Estimate τ^* by looking at the peak of the spectral density for the time series of frequency of price changes.
- Given N_a andτ^* we have an implies area outside threshold $|p| > \bar{x}$.
- We estimate $\tau^* \approx 15$ weeks and $N_a = 0.04$ per week, so area outside is: $Area \approx 0.60 = \tau^* \, N_a$
- Alternatively, every 15 weeks 40% the products goods change prices.

Estimates of τ^*

- Use weekly frequency of price changes (IRI)
- Took store in Chicago area with more goods and 573 weeks of data.
- HP filter weekly frequency for a class for each products (aisle).
- Estimate correlogram for weekly frequency
- Use kernel to estimate spectral density
- Look for the peak on the spectral density

Figure: Weekly Fraction of Price Changes

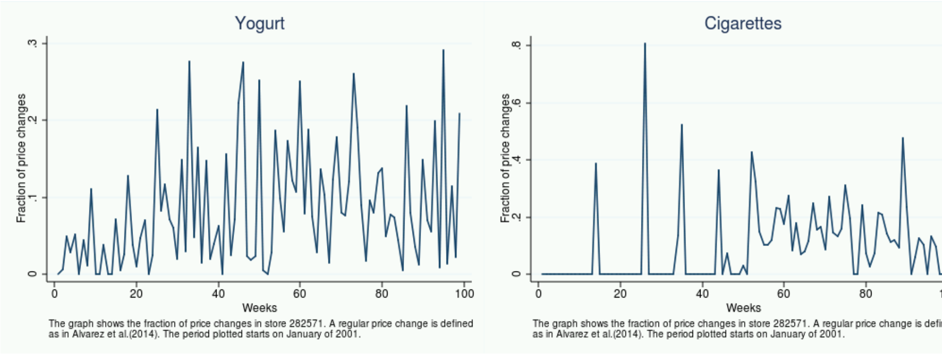


Figure: Power Spectral Density - Yogurt and Tooth Paste

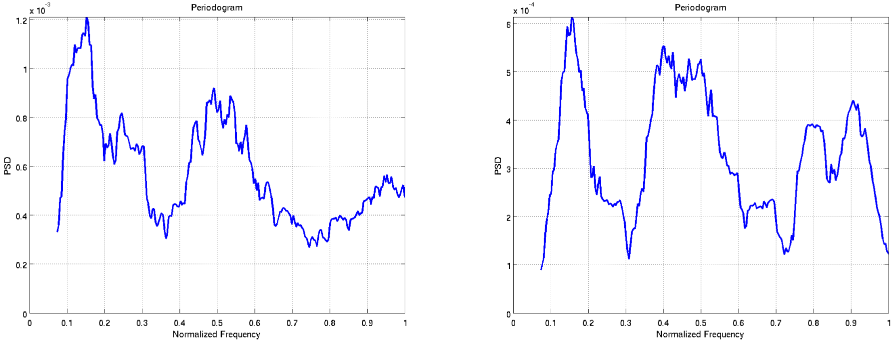
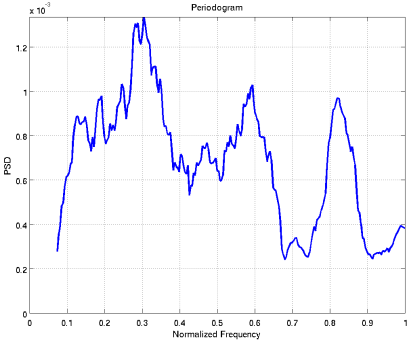


Figure: Power Spectral Density - Spaguetti Sauce and Peanut Butter



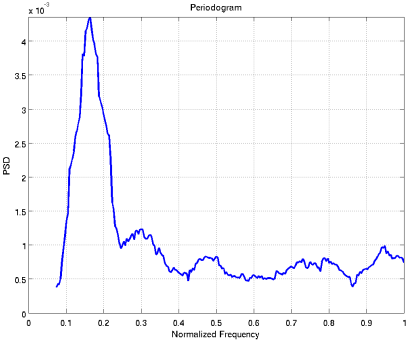
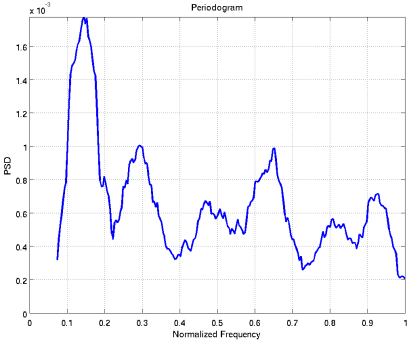


Figure: Power Spectral Density - Frozen Pizza and Cereal



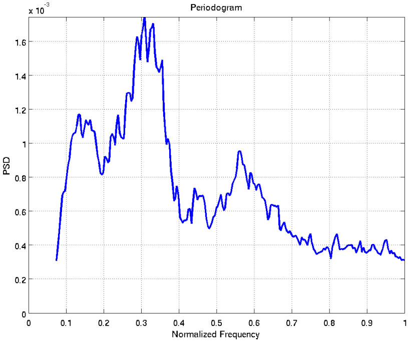


Table: Power Spectral Density - Interpretation of Normalized Frequency

$\tilde{x}$	**Period**
.03	52 weeks
.076	26 weeks
0.1	20 weeks
0.2	10 weeks
0.3	6.6 weeks
0.4	5 weeks

$\tilde{x}$	**Period**
0.5	4 weeks
0.6	3.3 weeks
0.7	2.8 weeks
0.8	2.5 weeks
0.9	2.2 weeks
1	2 weeks

Peak for most product types is around 0.15, or about 15 weeks.

Other comments

- Hazard rate of price changes
  - What is the hazard rate of individual price changes?
  - Hazard is positive only i in multiple of τ^* and increasing in time
- Add Calvo$^+$ to the model (random menu cost)
  - Kurtosis of individual price changes in the model will be closer to the data.
  - Very easy, as in the previous paper.
  - Better fit with positive frequency every week (see figure store 644)
- How to decide which products are in each “aisle"?
  Is the entire firm an “aisle"?