Fiscal policy coordination in currency unions at the effective lower bound

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January 2017

Abstract

According to the pre-crises consensus there are separate domains for monetary and fiscal stabilization in a currency union. While the common monetary policy takes care of union-wide fluctuations, fiscal policies should be tailored to meet country-specific conditions. This separation is no longer optimal, however, if monetary policy is constrained by an effective lower bound on interest rates. Specifically, we show that in this case there are benefits from coordinating fiscal policies across countries. By coordinating on a common fiscal stance, policy makers are able to stabilize union-wide activity and inflation while avoiding detrimental movements of a country's terms of trade.

Keywords: Currency union, fiscal policy, effective lower bound, coordination, EMU, terms-of-trade externality, optimal policy
JEL-Codes: E61, E62, F41

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1 Introduction

In the wake of the global financial crisis, fiscal policy staged a comeback as a stabilization tool. Figure 1 displays two rough measures of the discretionary fiscal stance, both for the US and the euro area. The left panel shows the change of the cyclically adjusted government budget deficit, measured in percentage-point changes relative to the pre-crisis year 2007. The right panel shows the level of government consumption relative to trend output. Both measures are indicative of an expansionary fiscal stance during the recession: deficits rose sharply after 2007, as did government spending. It appears, however, that fiscal stabilization has been used more timidly in the euro area: not only did deficits increase less than in the US, government spending was also raised relatively less, given its higher pre-crisis level.

One possible explanation is that euro-area fiscal policy is largely determined at the country level, rather than at the union level and, hence, there may have been a failure to coordinate fiscal stabilization across the member states of the euro area.[1] In line with this conjecture, there have been calls for stronger policy coordination, urging European governments to engineer a larger fiscal expansion during 2008–09 (see, for instance, Krugman, 2008). For the same reason, the shift to austerity in the euro area after 2010 may have been excessive, as argued by many observers (see, for instance, Cotarelli, 2012). Against this background, we ask whether fiscal-stabilization policies should be coordinated across the member states of a currency union with a view towards stabilizing area-wide activity and inflation.

According to the pre-crises consensus fiscal stabilization should be geared towards country-specific conditions, because the common monetary policy can take care of union-wide fluctuations (Beetsma and Jensen, 2005; Kirsanova et al., 2007; Galí and Monacelli, 2008).[2] The recent economic and financial crises have exposed a shortcoming of this paradigm: in a severe economic downturn monetary policy may be constrained by an effective lower bound (ELB) on nominal interest rates and thus be unable to stabilize fluctuations at the union level. Moreover, it is precisely under these circumstances that fiscal policy is very effective in stabilizing economic activity (Christiano et al., 2011; Woodford, 2011).

In our analysis we therefore explicitly account for the possibility that an ELB constrains

[1] To be sure, there has been an attempt to coordinate European fiscal stabilization policies, namely through the *European Economic Recovery Plan*, discussed and legislated in 2008–09. According to Cwik and Wieland (2011) the measures foreseen by the plan amounted to 1.04 and 0.86 percent of 2009 and 2010 GDP, respectively. Hence, they were considerably smaller than those due to US legislation under the *American Recovery and Reinvestment Act* which amounted to roughly 5 percent of GDP.

[2] Earlier contributions also allow for the possibility that the objectives of monetary and fiscal policy differ. This does not necessarily strengthen the case for coordination (Dixit and Lambertini, 2003). In fact, fiscal coordination may even be harmful (Beetsma and Bovenberg, 1998). Dixit and Lambertini (2001) offer some qualifications as well as further references. Schmidt (2013) studies coordinated monetary and fiscal stabilization in a closed economy when an effective bound on interest rates binds.

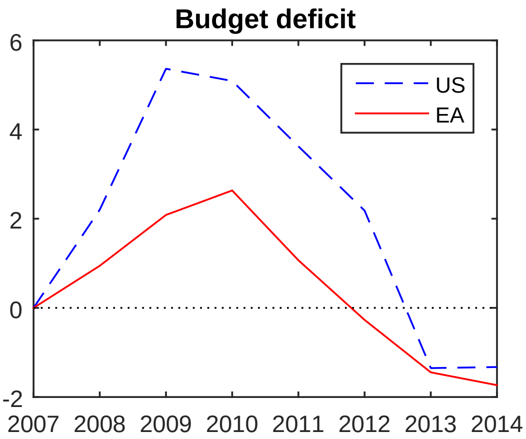

Figure 1: Euro area (EA): solid lines, United Staates (US): dashed lines; left panel: cyclically adjusted deficit, annual observations, change relative to 2007, measured in percentage points of potential output; right panel: government consumption of general government in units of trend output, quarterly observations; source: OECD Economic Outlook.

monetary policy. We do so within the framework of Galí and Monacelli (2008). It specifies a currency union which consists of a continuum of small open economies, each negligible in terms of aggregate outcomes. Yet as countries specialize in the production of a specific set of goods, domestic policies—if enacted unilaterally—will generally impact a country's terms of trade. In the absence of policy coordination, the optimal policy will therefore be conducted with a view towards its effect on the terms of trade. Instead, by coordinating on a common policy, countries can internalize this "terms-of-trade externality".[3] Hence, optimal policies will generally differ depending on whether there is coordination across countries or not.

In terms of country-specific policies, we focus on government spending. We assume that the government purchases only domestically produced goods, financed by lump-sum taxes levied on domestic households. We consider a representative household in each country which supplies labor and trades a complete set of state-contingent assets across countries. Its consumption basket includes goods produced in all countries of the union, but is biased towards domestically produced goods. Goods, in turn, are produced in a monopolistic competitive environment and firms are restricted in their ability to adjust prices. In "normal" (or pre-crisis) times monetary policy is able to perfectly stabilize inflation and output at the union level: there is no need for fiscal coordination across countries. We contrast this situation with a "crisis scenario" where monetary policy is unable to lower interest rates sufficiently in

[3]See Corsetti and Pesenti (2001), Benigno and Benigno (2003), De Paoli (2009) and Forlati (2015) for different perspectives on the terms-of-trade externality in the context of monetary policy.

response to a union-wide contractionary shock, because it is constrained by an ELB.[4]

We determine the optimal discretionary adjustment of government spending in the crisis scenario. Under coordination fiscal policies are set to maximize union-wide welfare. In the absence of coordination fiscal policy makers maximize country-specific welfare. We find that—in line with the conjecture above—countries provide too little stimulus at the ELB in the absence of coordination. Intuitively, local policymakers are keen to avoid the terms of trade appreciating too much with higher spending, as this lowers the demand for domestically produced goods at times of economic slack. Conversely, the increase of government spending is higher under coordination, because policy makers anticipate that the terms of trade remain unaffected by a policy response which is common across countries. At the same time, such a response is expected to boost union-wide inflation (rather than an individual country's terms of trade). This is desirable at the ELB, because expected inflation lowers the real interest rate. We illustrate that the fiscal *stimulus gap* due to the lack of coordination can be quantitatively significant.

Within our framework we recoup two results which have already been established in the literature, but are crucial to put our main result into perspective. First, we confirm an earlier finding of Turnovsky (1988) and Devereux (1991): absent cooperation policy makers choose *too high* a level of government spending in steady state. This is because governments seek to improve their country's terms of trade through purchases of domestically produced goods.[5] Hence, in steady state the terms-of-trade externality has the opposite effect than in the crisis scenario, because stronger terms of trade are beneficial in the long run, as the economy operates at full capacity.

Second, we also contrast government spending multipliers, that is, the percentage change of domestic output given a (possibly non-optimal) increase of government spending by one percent of GDP in the entire union and in the domestic economy only. In line with earlier work by Fahri and Werning (2016), we find that the multiplier is larger than unity in the first case, provided the ELB binds, but smaller than unity in the second case.[6] This result

[4]We abstract from non-conventional policies such as forward guidance (Eggertsson and Woodford, 2003) or credit policies by the central bank (see, e.g., Cúrdia and Woodford, 2011). These policies are arguably an imperfect substitute for conventional policies, if only because they are not very well understood and hence controversial (see, e.g., Rogoff, 2016). The effectiveness of forward guidance, in particular, appears to be limited (Giannoni et al., 2016).

[5]Epifani and Gancia (2009) find that this mechanism may account for the size of the public sector in open economies. In particular, their findings suggest that the terms-of-trade externality rather than a demand for insurance causes the public sector to grow with trade openness.

[6]Erceg and Lindé (2012) also compute spending multipliers for a small open economy. Assuming an exchange rate peg, they show that multipliers are always below unity. Assuming a specific scenario under which the ELB binds, they find that for multipliers to exceed unity prices need to be sufficiently flexible. Nakamura and Steinsson (2014), in turn, show that multipliers are high within a currency union when compared to the multiplier at the union level in the absence of a binding ELB constraint. Acconcia et al. (2014) find for Italian

obtains because a unilateral increase of government spending appreciates the terms of trade and thus crowds out private expenditure. Instead, the cooperative policy, common to all countries, raises expected inflation at the union level, thus crowding-in private expenditure at the ELB.[7]

Our analysis also relates to a number of other recent studies. Cook and Devereux (2011) study optimal fiscal policy in a two-country model when monetary policy is stuck at the ELB. However, they focus on the case of coordination, rather than on a possible coordination failure. Blanchard et al. (2016) calibrate a two-country model to capture key features of the euro area, notably of its core and periphery. They share our focus on the gains from cooperation. However, because their model is more complex than ours, their analysis is limited to numerical evaluations of an ad-hoc welfare criterion. Evers (2015) studies the performance of alternative fiscal arrangements in a quantitative model of a currency union. He finds that a centralized fiscal authority dominates a regime of fiscal transfers as well as a regime of decentralized fiscal decision making. Other work has focused on the coordination of debt and deficit policies in currency unions (Beetsma and Uhlig, 1999; Krogstrup and Wyplosz, 2010). We abstract from this aspect, as Ricardian equivalence obtains in our model. Moreover, we stress that our analysis disregards complications due to sovereign risk. However, both aspects are likely to further strengthen the case for coordination in currency unions stuck at the ELB (Corsetti et al., 2014).

The remainder of the paper is structured as follows. In Section 2 we describe the basic setup of the model. It also contrasts government spending multipliers at the union and the country level, once the ELB binds. In Section 3 we analyze the need for coordination by determining optimal government spending with and without coordination. Section 4 provides a quantitative assessment. Section 5 concludes.

2 Model

Our analysis is based on the model of Galí and Monacelli (2008). There is a currency union which consists of a continuum of countries, each a small open economy indexed by $i \in [0, 1]$. Each economy features a representative household, a continuum of monopolistically competitive firms and a fiscal authority. Monetary policy is conducted at the union level. We consider two dimensions which are absent in Galí and Monacelli (2008). First, we allow for the possi-

data that variations in local government spending have fairly strong output effects.

[7] An alternative perspective emphasizes monetary conditions: at the country level there is a de facto target for the price level, given by purchasing power parity. Any inflationary impulse due to fiscal policy thus triggers an offsetting deflationary tendency and causes the long term real interest rate to rise on impact (Corsetti et al., 2013b). At the union level, absent a price level target, the inflationary impulse due to higher government spending reduces real interest rates at the ELB.

bility that the ELB constrains monetary policy because of a union-wide contractionary shock. Second, we compute optimal fiscal policies when there is no coordination across countries.[8] Our exposition focuses on the model structure in terms of preferences and technology. In a second step, we state the linearized equilibrium conditions at the country and the union level. Readers may consult Galí and Monacelli (2008) for further details on the derivations.

2.1 Model structure

In what follows we briefly outline the problem of households, the fiscal authority, firms and monetary policy.

Households

A representative household in country i has preferences over private consumption, C_t^i, public consumption, G_t^i, and labor, N_t^i, given by

$$U(C_t^i, N_t^i, G_t^i) = (1-\chi)\log C_t^i + \chi \log G_t^i - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi},$$

where parameter $\chi \in (0,1)$ determines the relative weights of private and public consumption. $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. Private consumption is a composite of domestically produced goods, $C_{i,t}^i$, and imported goods, $C_{F,t}^i$:

$$C_t^i \equiv \frac{\left(C_{i,t}^i\right)^{1-\alpha}\left(C_{F,t}^i\right)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}.$$

Parameter $\alpha \in (0,1)$ measures the openness of the economy. Because country i has zero weight in the union, $\alpha < 1$ implies that there is home bias in consumption which accounts for deviations from purchasing power parity in the short run. Domestically produced goods are a CES basket of product varieties:

$$C_{i,t}^i \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \text{ with } \varepsilon > 1. \tag{1}$$

Here $C_{i,t}(j)$ denotes consumption of variety $j \in [0,1]$ in country i. Parameter $\varepsilon > 1$ denotes the elasticity of substitution between different varieties of goods produced within each country. Imported goods, in turn, are defined as follows:

$$C_{F,t}^i \equiv \exp \int_0^1 c_{f,t}^i df,$$

[8] Forlati (2009) also analyzes optimal fiscal policy in the absence of coordination within the Galí-Monacelli model. Her focus is on the interaction of monetary and fiscal policy without considering an ELB.

with $c^i_{f,t} \equiv \log \mathcal{C}^i_{f,t}$ and $f \in [0,1]$. The index $\mathcal{C}^i_{f,t}$ is defined analogously to (1), with an appropriate normalization.[9]

Given the definitions above, minimizing expenditures gives rise to demand functions for product varieties. For instance, domestic demand for generic good j is given by

$$C^i_{i,t}(j) = \left(\frac{P^i_t(j)}{P^i_t}\right)^{-\varepsilon} C^i_{i,t},$$

where $P^i_t(j)$ is the price of good j and $P^i_t \equiv \left(\int_0^1 P^i_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$ is the domestic (producer) price index. Similarly, country-i demand for a generic country-f good j is given by

$$C^i_{f,t}(j) = \left(\frac{P^f_t(j)}{P^f_t}\right)^{-\varepsilon} C^i_{f,t}.$$

Here $P^f_t(j)$ and P^f_t have the same interpretation as the domestic counterparts. The optimal allocation between domestic and foreign goods requires

$$C^i_t = (1-\alpha)\left(\frac{P^i_t}{P^i_{c,t}}\right)^{-1} C^i_t, \qquad C^i_{F,t} = \alpha \left(\frac{P^*_t}{P^i_{c,t}}\right)^{-1} C^i_t,$$

where $P^*_t \equiv \exp \int_0^1 p^f_t df$ is the union-wide price index. The consumer price index (CPI) is given by $P^i_{c,t} \equiv (P^i_t)^{1-\alpha}(P^*_t)^{\alpha}$. In the following we focus on the producer price index, P^i_t, which is related to the CPI according to $P^i_t = P^i_{c,t}(S^i_t)^{\alpha}$, where $S^i_t \equiv P^*_t / P^i_t$ denotes the terms of trade.

Households trade a complete set of state-contingent securities which provides insurance against country-specific shocks.[10] They maximize expected discounted lifetime utility subject to a sequence of budget constraints:

$$\begin{aligned} \max_{\{C^i_t, N^i_t, A^i_t\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(C^i_t, N^i_t, G^i_t) \\ \text{s.t.} \quad & P^i_{c,t} C^i_t + E_t\{Q_{t,t+1} A^i_{t+1}\} \leq A^i_t + W^i_t N^i_t + \mathcal{P}^i_t - T^i_t. \end{aligned}$$

where A^i_t denotes the portfolio of nominal assets and $Q_{t,t+1}$ is the nominal stochastic discount factor (common across countries). Ponzi schemes are not permitted. W^i_t is the nominal wage and $\mathcal{P}^i_t$ are firm profits, rebated to households in a lump-sum fashion. T^i_t are lump-sum taxes. Parameter $\beta \in (0,1)$ is the subjective discount factor.

[9] See Galí and Monacelli (2015). Without normalization either consumption of foreign goods or total consumption goes to zero (Hellwig, 2015).

[10] For instance, idiosyncratic technology shocks as in Galí and Monacelli (2008). As we analyze optimal policy in response to an aggregate shock that pushes the currency union at the ELB we abstract from country-specific shocks in our analysis.

Fiscal authority

Public consumption is composed of domestically produced goods as in (1) and the fiscal authority allocates expenditures in a cost minimizing manner. The resulting demand function for a generic good j is given by:

$$G_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon} G_t^i,$$

where G_t^i is aggregate expenditure, the level of which remains to be determined below. Taxes adjust to balance the budget in each period:

$$T_t^i = P_t^i G_t^i + \tau^i W_t^i N_t^i.$$

where τ^i is a (constant) employment subsidy paid to domestic firms. If set appropriately it ensures the efficiency of the steady state under monopolistic competition.

Firms

In each country, there is a continuum of monopolistically competitive firms, each of which produces a differentiated good $Y_t^i(j)$. These goods are traded across countries and the law of one price is assumed to hold. Firms cannot adjust their price $P_t^i(j)$ every period. Instead, as in Calvo (1983) they may reset prices in a given period with probability $1-\theta$, while their current price remains in effect with probability $\theta \in (0,1)$. The probability of resetting the price is independent of a firm's last adjustment. Firms hire labor $N_t^i(j)$ and produce with a linear technology $Y_t^i(j) = N_t^i(j)$ in order to satisfy the level of demand at a given price. The objective of a generic firm $j \in [0,1]$ is to maximize discounted, expected nominal payoffs taking the demand for its product into account:

$$\max_{\bar{P}_t^i(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k}^i(j)(\bar{P}_t^i(j) - (1-\tau^i)W_{t+k}^i) \right\}$$
$$\text{s.t.} \quad Y_{t+k}^i(j) = \left(\frac{\bar{P}_t^i(j)}{P_{t+k}^i}\right)^{-\varepsilon} Y_{t+k}^i,$$

where $\bar{P}_t^i(j)$ is the optimal price, set in period t.

Monetary policy

Monetary policy is conducted at the union level. The policy instrument is the nominal interest rate, that is, the yield on a nominally riskless one-period discount bond: $1 + i_t^* \equiv \frac{1}{E_t Q_{t,t+1}}$. The objective of monetary policy is to maintain price stability, that is, zero inflation at the union level.[11] Importantly, monetary policy may be constrained by an ELB. Specifically, in what follows we assume that $i_t^* \geq 0$, that is, we assume the effective lower bound to be zero. While the actual lower bound is arguably somewhat below zero, this is of little consequence in the context of our analysis. Below we specify an interest-rate rule which implements price stability subject to the ELB constraint.

2.2 Equilibrium conditions for approximate model

We consider a log-linear approximation to the optimality and market-clearing conditions around a symmetric, zero-inflation steady state. We use hats to denote log-deviations of a variable from its steady-state value. For a generic variable X_t we define $x_t \equiv \log X_t$ and $\hat{x} = \log(X_t/X)$. Union-wide variables are obtained by integrating over all countries in the union: $\hat{x}_t^* = \int_0^1 \hat{x}_t^i di$.
First, goods-market clearing and integrating over all goods gives for country i

$$\hat{y}_t^i = (1-\gamma)(\hat{c}_t^i + \alpha s_t^i) + \gamma \hat{g}_t^i. \tag{2}$$

Parameter γ denotes the steady-state ratio of government consumption to output. The above equation links domestic output $\hat{y}_t^i$ to domestic consumption $\hat{c}_t^i$, the terms of trade s_t^i and domestic government spending $\hat{g}_t^i$. Further, the assumption of complete markets gives rise to the following risk sharing condition:

$$\hat{c}_t^i = \hat{c}_t^* + (1-\alpha)s_t^i. \tag{3}$$

Combining it with (2) gives

$$\hat{y}_t^i = \gamma \hat{g}_t^i + (1-\gamma)\hat{c}_t^* + (1-\gamma)s_t^i. \tag{4}$$

Equation (4) is an equilibrium condition at the country level which replaces the conventional dynamic IS-equation (which still operates at the union level, see below). Integrating equation (4) over all countries $i \in [0,1]$ and noting that $\int_0^1 s_t^i di = 0$ leads to the union-wide market clearing condition

$$\hat{y}_t^* = \gamma \hat{g}_t^* + (1-\gamma)\hat{c}_t^*. \tag{5}$$

The second equilibrium condition at the country level accounts for the price-setting behavior of firms and the law of motion for the price level. Specifically, we obtain the following variant of the New Keynesian Phillips curve:

$$\pi_t^i = \beta E_t\{\pi_{t+1}^i\} + \lambda \left(\frac{1}{1-\gamma} + \varphi\right) \hat{y}_t^i - \frac{\lambda\gamma}{1-\gamma}\hat{g}_t^i, \tag{6}$$

[11] In the context of our model this is the optimal discretionary policy under fiscal coordination.

with $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ and where $\pi_t^i = p_t^i - p_{t-1}^i$ denotes the inflation rate. From the definition of the terms of trade it follows that

$$\pi_t^i = \pi_t^* - s_t^i + s_{t-1}^i. \tag{7}$$

Equations (4), (6) and (7) characterize the equilibrium in the small open economy given a process for government spending and union-wide variables.

The union-wide equilibrium conditions are given by an aggregate Phillips curve:

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda \left(\frac{1}{1-\gamma} + \varphi \right) \hat{y}_t^* - \frac{\lambda\gamma}{1-\gamma} \hat{g}_t^*, \tag{8}$$

which is obtained by integrating over the country-specific Phillips curves (6) and by a dynamic IS curve:

$$\hat{y}_t^* = E_t\{\hat{y}_{t+1}^*\} - (1-\gamma)(i_t^* - E_t\{\pi_{t+1}^*\} - r_t) - \gamma E_t\{\hat{g}_{t+1}^*\} + \gamma \hat{g}_t^* \tag{9}$$

with $r_t \equiv -\log\beta - \Delta_t$. As in Woodford (2011), Δ_t denotes a spread between the interest rate set by the central bank and the one relevant for private sector decisions. It reflects frictions in financial intermediation which we do not model explicitly, but permit to vary exogenously.[12] If this spread becomes large enough, monetary policy becomes constrained by the ELB. In what follows, we thus assume that the conduct of monetary policy can be described by the following rule

$$i_t^* = \max\{r_t + \phi_\pi \pi_t^*, 0\}. \tag{10}$$

By following this rule monetary policy fully stabilizes inflation and output at the union level (as long as $\hat{g}_t^* = 0$), unless the ELB binds.

Effective-lower-bound scenario. In our analysis below, we consider a scenario where the ELB binds because the interest rate spread increases temporarily. Specifically, as in Woodford (2011), we assume a Markov structure for r_t. It declines temporarily to a value $r_L < 0$. The shock remains operative with probability μ and is sufficiently large for the ELB to become binding. Hence, (10) implies that $i_t^* = 0$ for as long as the shock lasts, independently on the conduct of fiscal policy.[13] With probability $1-\mu$ the spread disappears (and thus the whole economy) returns permanently to the steady state. Moreover, defining $\kappa \equiv \lambda\left(\frac{1}{1-\gamma} + \varphi\right)$, we impose the parametric restriction $(1-\mu)(1-\beta\mu) > (1-\gamma)\mu\kappa$ for the equilibrium to be uniquely determined (Woodford, 2011).

[12] Cúrdia and Woodford (2015) provide a microfoundation in a model which accounts for household heterogeneity and borrowing and lending across households.

[13] Schmidt (2013) and Erceg and Lindé (2014) consider endogenous exit from the ELB due to fiscal-policy measures. We assume instead that the decline of r_t is sufficiently large for the ELB to remain binding also in the presence of optimal fiscal stabilization.

Definition of equilibrium. *Given initial conditions* (s_{-1}) *as well as* $\{r_t\}_{t=0}^{\infty}$ *an equilibrium is a collection of*

1. *country-specific stochastic processes* $\{\hat{y}_t^i, \pi_t^i, s_t^i\}_{t=0}^{\infty}$ *for all* $i \in [0,1]$

2. *union-wide stochastic processes* $\{\hat{y}_t^*, \pi_t^*\}_{t=0}^{\infty}$ *with* $\hat{y}_t^* = \int_0^1 \hat{y}_t^i di$, $\pi_t^* = \int_0^1 \pi_t^i di$

such that for given $\{\hat{g}_t^i\}_{t=0}^{\infty}$ *for all* $i \in [0,1]$ *with* $\hat{g}_t^* = \int_0^1 \hat{g}_t^i di$ *and the path for the nominal interest rate* $\{i_t^*\}_{t=0}^{\infty}$ *determined by* (10)

3. *equilibrium conditions (4), (6) and (7) are satisfied for each country i and*

4. *equilibrium conditions (8) and (9) are satisfied on the union level.*

For later purposes, we note that the equilibrium conditions imply the following second order stochastic difference equation for the terms of trade (see Galí and Monacelli, 2005)

$$s_t^i = \omega s_{t-1}^i + \omega\beta E_t\{s_{t+1}^i\} - \omega\lambda\varphi\gamma(\hat{g}_t^i - \hat{g}_t^*),$$

where $\omega \equiv \frac{1}{1+\beta+\lambda[1+\varphi(1-\gamma)]} \in [0, \frac{1}{1+\beta})$. The above equation has a unique stable solution

$$s_t^i = \delta s_{t-1}^i + \delta\lambda\varphi\gamma \sum_{k=0}^{\infty} (\beta\delta)^k E_t\{\hat{g}_{t+k}^* - \hat{g}_{t+k}^i\}, \tag{11}$$

with $\delta \equiv \frac{1-\sqrt{1-4\beta\omega^2}}{2\omega\beta} \in (0,1)$.

2.3 Impact multipliers: union-wide vs country-specific fiscal impulse

In this section, to set the stage for our main results in Section 3, we solve for the government spending multiplier on output. That is, we determine by how much country-specific output changes, given an increase of government consumption by one percent of output. Our focus is on how the multiplier differs depending on whether there is a union-wide or a country-specific variation of government consumption. As a union-wide fiscal impulse impacts the individual countries symmetrically, this scenario is equivalent to the closed-economy setting in Woodford (2011). Instead, a country-specific fiscal impulse impacts domestic output directly, but also indirectly via the terms of trade. This scenario is thus equivalent to the small-open-economy settings in Corsetti et al. (2013b) and Fahri and Werning (2016). We briefly revisit their results within our framework.

Consider first the union-wide fiscal impulse in the ELB scenario. We assume that government spending is increased in every country by the same amount as long as the ELB remains binding. In this case, given the assumptions spelled out above, union-wide variables take a constant value x_L^*, as long as the shock persists. In this case, the union-wide Phillips curve and the IS equation simplify to

$$\pi_L^* = \frac{1}{1-\beta\mu}\kappa(\hat{y}_L^* - \frac{\bar{\sigma}\gamma}{\bar{\sigma}+\varphi}\hat{g}_L^*), \tag{12}$$

$$(1-\mu)(\hat{y}_L^* - \gamma\hat{g}_L^*) = (1-\gamma)\mu\pi_L^* + (1-\gamma)r_L, \tag{13}$$

with $\bar{\sigma} \equiv \frac{1}{1-\gamma}$.

We solve the above system for $\hat{y}_L^*$ as a function of r_L and $\hat{g}_L^*$. This gives:

$$\hat{y}_L^* = \frac{(1-\gamma)(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-(1-\gamma)\mu\kappa}r_L + \frac{(1-\mu)(1-\beta\mu)\gamma-(1-\gamma)\mu\kappa\frac{\gamma\bar{\sigma}}{\bar{\sigma}+\varphi}}{(1-\mu)(1-\beta\mu)-(1-\gamma)\mu\kappa}\hat{g}_L^*. \tag{14}$$

In order to determine the multiplier, we divide the derivative of $\hat{y}_L^*$ with respect to $\hat{g}_L^*$ by the steady-state share of government spending, γ:

$$\frac{1}{\gamma}\frac{\partial\hat{y}_L^*}{\partial\hat{g}_L^*} = \frac{(1-\mu)(1-\beta\mu)-(1-\gamma)\mu\kappa\frac{\bar{\sigma}}{\bar{\sigma}+\varphi}}{(1-\mu)(1-\beta\mu)-(1-\gamma)\mu\kappa} \geq 1.$$

At the union level, we thus find that the multiplier is bound from below by unity (Woodford, 2011). Intuitively, higher government spending reduces real interest rates at the ELB, because the expected inflationary impact of higher spending is not matched by higher nominal interest rates. Hence, private-sector spending is crowded in.

We now turn to the effect of a country-specific fiscal impulse. In this case, for simplicity but without loss of generality we set union-wide variables to zero and assume that government spending in country i follows a two-state Markov switching process. Initially, government spending exceeds its steady state level $\hat{g}_L^i > 0$; it does so with probability μ in the next period too and returns to steady state with probability $1-\mu$.

Specifically, equations (4) and (11), evaluated in the impact period of the spending increase read as follows

$$\hat{y}_1^i = \gamma\hat{g}_L^i - (1-\gamma)p_1^i$$

$$p_1^i = \frac{\delta\lambda\varphi\gamma}{1-\beta\delta\mu}\hat{g}_L^i.$$

Combining both equations, we obtain the government spending multiplier in the impact period.[14] It is given by

$$\frac{1}{\gamma}\frac{\partial\hat{y}_1^i}{\partial\hat{g}_L^i} = 1-(1-\gamma)\underbrace{\frac{\delta\lambda\varphi}{1-\beta\delta\mu}}_{\geq 0} \leq 1.$$

[14]Because a country-specific fiscal impulse impacts the terms of trade, the output effect of government spending changes over time even though the size of the impulse does not. We focus on the impact effect.

The upper bound of unity is reached when prices are completely sticky ($\lambda \to 0$). To the extent that prices are somewhat flexible, private-sector spending at the country level is crowded out by higher government consumption. Its inflationary impact appreciates the terms of trade which, in turn, calls for reduced consumption in country i, see equation (3). Equivalently, (relative) purchasing power parity requires that the price level reverts back to its pre-shock level in the long run. Given unchanged nominal interest rates in the currency union, future deflation induces long-term real interest rates to rise on impact. Still, the crowding-out effect of country-specific stimulus in a currency union is limited relative to when the country operates a flexible exchange rate system (see, for further discussion and evidence, Corsetti et al., 2013b; Born et al., 2013).

Taken together, we obtain the following ranking of the government spending multiplier on country-specific output, considering a union-wide and country-specific spending increase, respectively:

$$\frac{1}{\gamma}\frac{d\hat{y}_1^i}{d\hat{g}_L^i} \leq 1 \leq \frac{1}{\gamma}\frac{d\hat{y}_L^*}{d\hat{g}_L^*}.$$

Fahri and Werning (2016) obtain this result as a closed-form solution of the continuous-time version of the New Keynesian model.

3 Optimal policy

We now turn to optimal fiscal policy. In particular, we distinguish between a scenario of coordination and one without, both with regards to the steady state and to the ELB scenario. In each case, policy makers chose government consumption in order to maximize household welfare.

3.1 Steady state

We consider optimal fiscal policy in the steady state first. We compute the steady state as the solution to the social planer problem and discuss how it can be decentralized in a symmetric, zero-inflation steady state.

Under coordination the social planner (of the union) maximizes union-wide welfare subject to the production function, the goods-market-clearing condition and the risk-sharing condition in every country $i \in [0, 1]$. The planner also accounts for the effect of country-specific consumption levels on union-wide consumption. Formally, we have

$$\max \int_0^1 \left((1-\chi)\log C^i + \chi \log G^i - \frac{\left(N^i\right)^{1+\varphi}}{1+\varphi} \right) di \tag{15}$$

$$\begin{aligned} \text{s.t.} \quad & Y^i = N^i \\ & Y^i = C^i (S)^{\alpha} + G^i \\ & C^i = C^*(S)^{1-\alpha} \\ & C^* = \int_0^1 C^i di. \end{aligned}$$

In addition, optimality requires that varieties are produced and consumed in equal quantities in a given country. Solving the planner problem gives rise to the following steady state relations (for each country $i \in [0,1]$), see Appendix A:

$$\gamma^{Coord} \equiv \left(\frac{G}{Y}\right)^{Coord} = \chi; \quad Y^{Coord} = 1. \tag{16}$$

The social planer solution can be decentralized as a symmetric, zero-inflation steady state by letting the government provide public goods according to (16) and by choosing the labor subsidy which offsets distortions due to monopolistic competition (see Galí and Monacelli, 2008):

$$\tau^{Coord} = \frac{1}{\varepsilon}.$$

We now turn to the case without coordination. Here, the social planner (in a given country i) maximizes domestic welfare only, subject to the production function, the market-clearing condition and the risk-sharing condition. The planner does not take effects on union-wide consumption into account, but takes consumption in the rest of the union C_t^* as given. Formally, we have

$$\max U(C^i, N^i, G^i) = (1-\chi)\log C^i + \chi \log G^i - \frac{\left(N^i\right)^{1+\varphi}}{1+\varphi} \tag{17}$$

$$\begin{aligned} \text{s.t.} \quad & Y^i = N^i \\ & Y^i = C^i (S)^{\alpha} + G^i \\ & C^i = C^*(S)^{1-\alpha}. \end{aligned}$$

Again, optimality requires that varieties are produced and consumed in equal quantities in a given country. In a symmetric Nash equilibrium, optimality requires the following to hold in steady state in every country $i \in [0,1]$, see Appendix B.1:

$$\begin{aligned} \gamma^{Nash} \equiv \left(\frac{G}{Y}\right)^{Nash} &= \frac{\chi}{(1-\alpha)(1-\chi)+\chi} \in (0,1) \\ Y^{Nash} &= \left[(1-\alpha)(1-\chi)+\chi\right]^{\frac{1}{1+\varphi}}. \end{aligned} \tag{18}$$

Comparing the outcome under coordination and without, we observe that the government-consumption-to-output ratio is higher in the latter case: $\gamma^{Nash} > \gamma^{Coord}$. Furthermore the level of government spending without coordination exceeds the level under coordination: $G^{Nash} > G^{Coord}$, even though output is lower $Y^{Nash} < Y^{Coord}$, see also Appendix B.1.
This confirms earlier findings by Turnovsky (1988) and Devereux (1991) according to which government consumption without coordination accounts for an excessively large share of output. Intuitively, each government tries to improve the domestic terms of trade by increasing domestic demand. In a symmetric Nash equilibrium, however, the terms of trade are equal to unity. Government consumption is higher and output is lower relative to the case of coordination. Because of the terms-of-trade externality, the Nash steady state is inefficient: welfare is lower than in case of coordination.
The social planer solution in the absence of coordination can be decentralized as a symmetric, zero-inflation steady state by letting the government provide public goods according to (18) and by choosing the following labor subsidy (see Appendix B.2):

$$1 - \tau^{Nash} = \left(1 - \frac{1}{\varepsilon}\right)(1 - \alpha)^{-1}.$$

Again, zero-inflation ensures that the same amount of each variety is produced and consumed.

3.2 Effective lower bound

In order to determine the optimal policy at the ELB, we pursue a linear-quadratic approach. First, we approximate household welfare up to second order around the steady state with and without coordination. Galí and Monacelli (2008) provide such an approximation for the case of coordination. We provide details on the derivation in the absence of coordination in Appendix C.[15] Second, we determine the optimal discretionary fiscal policy in the dynamic model approximated around each steady state. For this purpose, we maximize the welfare functions subject to the equilibrium conditions.
Consider first the case of coordination. Here we focus on the symmetric solution, that is, $x_t^i = x_t^*$ for all $i \in [0, 1]$, because we analyze the effects of a union-wide shock and assume that countries are identical. Under coordination the single policymaker maximizes union-wide welfare by choosing government consumption in a discretionary way subject to the New Keynesian Phillips curve, (8), and an inequality constraint which consolidates the dynamic IS equation, (9) and the interest-rate rule (10). Hence, optimization is subject to the ELB.

[15] In the absence of coordination there are linear terms in a second-order approximation to household utility. We rely on the approach of Benigno and Woodford (2006) and substitute for these terms using a second-order approximation to the market-clearing condition.

Formally, assuming discretionary policy making, the optimization problem is given by

$$\begin{aligned}
&\max_{\pi_t^*,\hat{y}_t^*,\hat{g}_t^*} \mathbb{W}_t^* \simeq -\frac{1}{2}\left(\frac{\varepsilon}{\lambda}(\pi_t^*)^2 + (1+\varphi)(\hat{y}_t^*)^2 + \frac{\gamma^{Coord}}{1-\gamma^{Coord}}(\hat{g}_t^* - \hat{y}_t^*)^2\right) \\
&\text{s.t.} \quad \pi_t^* = \lambda\left(\frac{1}{1-\gamma^{Coord}} + \varphi\right)\hat{y}_t^* - \frac{\lambda\gamma^{Coord}}{1-\gamma^{Coord}}\hat{g}_t^* + \nu_{0,t}^* \\
&\qquad\ \hat{y}_t^* \leq \gamma^{Coord}\hat{g}_t^* + \nu_{1,t}^*,
\end{aligned} \tag{19}$$

where $\nu_{0,t}^*$ and $\nu_{1,t}^*$ collect expectation terms which are beyond the control of the policy maker under discretion. In Appendix D we show that the solution to (19) requires $\pi_t^* = \hat{y}_t^* = \hat{g}_t^* = 0$ as long as the monetary authority is not constrained by the ELB. Hence, in this case the economy is perfectly stabilized at the steady state—and optimal government spending at its steady-state level. When the ELB is binding, $\pi_t^* = \hat{y}_t^* = \hat{g}_t^* = 0$ is no longer feasible. In that case we find that optimal government spending is characterized by the following condition:

$$\pi_t^{*,Coord} + \frac{1}{\varepsilon}\hat{y}_t^{*,Coord} = -\psi_g^{Coord}\,\hat{g}_t^{*,Coord}, \tag{20}$$

where $\psi_g^{Coord} \equiv \frac{1}{\varepsilon\varphi} > 0$ (see case 2 in Appendix D). Intuitively, as long as output and inflation drop in the ELB scenario, equation (20) calls for an increase of government spending. The following proposition states the solution for optimal government spending.

Proposition 1. *Given the effective-lower-bound scenario under consideration (see Section 2.2), the solution for the optimal fiscal response under coordination is given by:*

$$\hat{g}_L^{*,Coord} = -\Theta^{Coord} r_L > 0,$$

with $\Theta^{Coord} \equiv \frac{(1-\gamma^{Coord})(\kappa^{Coord}\varepsilon + (1-\beta\mu))}{\psi_g^{Coord}\varepsilon[(1-\mu)(1-\beta\mu)-(1-\gamma^{Coord})\mu\kappa^{Coord}] + (1-\mu)\gamma^{Coord}\lambda\varphi\varepsilon + \gamma^{Coord}[(1-\mu)(1-\beta\mu)-\mu\lambda]} > 0$ *and* $\kappa^{Coord} \equiv \lambda\left(\frac{1}{1-\gamma^{Coord}} + \varphi\right)$.

Proof. See Appendix F. ■

Hence, we find that it is optimal to raise government spending under coordination, once the ELB binds. Our result is in line with Woodford (2011), because in the present context the currency union under coordination is isomorphic to his closed-economy model.

We now turn to optimal government spending in the absence of coordination. In this case, policy choices may differ across countries from an ex-ante perspective and, hence, are expected to impact a country's terms of trade. Given price stickiness, the terms of trade adjust sluggishly in a currency union, while their long-run value is determined by purchasing power parity. As a result the (lagged) terms of trade are an endogenous state variable and the policy problem is inherently dynamic in the absence of coordination—even under discretion. In this case, even though a policy maker may not directly steer private-sector expectations, current policy decisions impact expectations indirectly via endogenous state variables—an effect which is internalized by the policy maker (see, e.g. Svensson, 1997).

We further note that, since the policy maker takes union-wide variables as given, including the nominal interest rate, the union-wide IS curve is not a constraint to the decision maker and neither is the ELB. Instead, optimization is subject to the market-clearing condition, (4), the country-specific New Keynesian Phillips curve, (6), and the evolution of the terms of trade, (7). Specifically, under discretion the optimization problem is given by

$$V(s^i_{t-1}, \pi^*_t, \hat{c}^*_t) = \max_{\pi^i_t, \hat{y}^i_t, \hat{g}^i_t, s^i_t} \left[-\frac{1}{2} \left(\frac{\varepsilon}{\lambda} (\pi^i_t)^2 + (1+\varphi)(\hat{y}^i_t)^2 + \frac{\gamma}{1-\gamma} \left(\hat{g}^i_t - \hat{y}^i_t \right)^2 \right) + \beta E_t V(s^i_t, \pi^*_{t+1}, \hat{c}^*_{t+1}) \right] \tag{21}$$

$$\begin{aligned}
\text{s.t.} \quad & \hat{y}^i_t = \gamma \hat{g}^i_t + (1-\gamma)\hat{c}^*_t + (1-\gamma) s^i_t \\
& \pi^i_t = \beta E_t\{\pi^i_{t+1}\} + \lambda \left(\frac{1}{1-\gamma} + \varphi \right) \hat{y}^i_t - \frac{\lambda\gamma}{1-\gamma} \hat{g}^i_t \\
& \pi^i_t = \pi^*_t - s^i_t + s^i_{t-1} \\
& \text{and } E_t \pi^i_{t+1} \text{ given.}
\end{aligned}$$

In the expression above V is the value function. The solution to (21) requires the following (consolidated) first-order condition to be satisfied (see Appendix E):

$$-\lambda\beta E_t \frac{\partial V(s^i_t, \pi^*_{t+1}, \hat{c}^*_{t+1})}{\partial s^i_t} + \beta \left(\frac{1}{\varphi} \hat{g}^i_t + \hat{y}^i_t \right) \frac{\partial E_t \pi^i_{t+1}}{\partial s_t} + \frac{1}{\varepsilon} \hat{y}^i_t + \pi^i_t = -\psi^{Nash}_g \hat{g}^i_t \tag{22}$$

with $\psi^{Nash}_g \equiv \frac{1}{\varepsilon\varphi} (\lambda\varphi + (1+\lambda))$.

To develop some intuition, it is instructive to contrast optimality condition (22) with the one derived under coordination, eq. (20). For this purpose we abstract in a first step from the dynamic terms on the left of eq. (22). We observe that for a given drop of output and inflation in the ELB scenario under consideration, the optimal policy response entails a smaller increase of government spending than in case of coordination, since $\psi^{Nash}_g > \psi^{Coord}_g$. Intuitively, in the absence of coordination, a local policy maker anticipates that higher government spending appreciates the terms of trade which, in turn, lowers the demand for domestic goods. This effect is absent when government spending is raised simultaneously in all countries under coordination. A non-cooperative policy maker will therefore tend to opt for less fiscal stimulus.[16] The following proposition establishes this formally for the special case which eliminates the dynamic terms in eq. (22).

[16] Absent commitment there are also benefits from coordinating fiscal policy measures *intertemporally*. In

Proposition 2. *Consider the effective-lower-bound scenario and a symmetric equilibrium, while $\beta \to 0$. In this case optimal government spending is raised less without coordination than with coordination (in percentage terms of steady state spending). Formally, we have*

$$\hat{g}_L^{*,Nash} < \hat{g}_L^{*,Coord}.$$

Proof. See Appendix G. ■

In the general case for $\beta \in (0,1)$, the optimal policy also reflects the fact that the terms of trade operate as an endogenous state variable. The first term on the left of eq. (22) captures the effect that, all else equal, stronger terms of trade (that is, a lower s_t) are expected to persist and to reduce expected future welfare when foreign demand and foreign inflation are weak (as in the ELB scenario). To the extent that higher government spending appreciates the terms of trade, there is thus an additional incentive to opt for less spending in the absence of coordination. This effect reinforces the ordering established in Proposition 2.

Turning to the second term on the left of eq. (22), note that stronger terms of trade (that is, a lower s_t) reduce inflation expectations, because, as they persist, they will raise the purchasing power of workers and induce downward pressure on wages and inflation (that is, $\partial E_t \pi_{t+1}^i / \partial s_t > 0$). Via the Phillips curve, lower expected inflation reduces inflation today. This dynamic terms-of-trade channel attenuates the appreciation of the terms of trade in response to higher government spending (and output) and makes a fiscal stimulus in the absence of cooperation relatively more attractive.[17] Yet this dynamic channel does not overturn the ordering established in Proposition 2 because it merely dampens the appreciation of the terms of trade.

Overall, we thus find that the terms of trade are crucial for optimal policy design, not only in steady state, but also off steady state. Intuitively, absent cooperation there is excessive government consumption in steady state, because better terms of trade are beneficial in the long run, as the economy operates at full capacity. In the short-run, local policy makers are keen to avoid the terms of trade appreciating too much with higher spending, as it reduces the demand for domestic goods in times of economic slack.

particular, Schmidt (2016) shows in a closed-economy setup that it may be desirable to appoint a fiscally activist policy maker, because anticipation of aggressive fiscal expansions at the ELB mitigates the adverse implications of the ELB today.

[17] The sluggish adjustment of the terms of trade has been identified as a key determinant of the macroeconomic adjustment mechanism in monetary unions. Benigno (2004) and Pappa (2004) stress how it distorts the adjustment. Groll and Monacelli (2016), in contrast, relate it to the "intrinsic benefits of monetary unions" in the absence of commitment.

Table 1: Parameter values

χ	0.148	Public consumption-GDP ratio
α	0.2874	Import-share in steady state
β	0.99	Discount factor
θ	0.925	Degree of price stickiness
ε	6	Elasticity of substitution
φ	4	Inverse of Frisch elasticity of labor supply
ϕ_π	1.5	Taylor coefficient
r_L	-0.01	ELB scenario

4 Quantitative assessment

In what follows we explore to which extent our result matters quantitatively. We assign parameter values by targeting observations for the euro area and the US for the period 1999–2006. We treat the US as a benchmark for a currency union in which government spending is set cooperatively. For the euro area, in contrast, we assume that government expenditures are set non-cooperatively.[18]

A period in the model corresponds to one quarter. We set $\chi = 0.148$ in order to match the average share of exhaustive government consumption relative to GDP in the US (see Figure 1 above). To match the share of government consumption in the euro area which is equal to 0.196, we set the openness parameter α to 0.2874, see equation (18).[19] Further, we set the time-discount factor β to 0.99 and $\theta = 0.925$. Such a high degree of price stickiness appears to be justified in light of the inflation dynamics observed in the context of the crisis (Corsetti et al., 2013a). Moreover, as we illustrate by means of a sensitivity analysis below, understating the extent of price rigidity biases results in favor of fiscal policy coordination. A high degree of price rigidity is thus a conservative choice. We set the elasticity of substitution among varieties to $\varepsilon = 6$. This implies an average markup of 20 percent. We assume $\varphi = 4$, such that the Frisch elasticity is moderate, in line with recent evidence (Chetty et al., 2011). We explore below to what extent results vary with β, θ, φ and α. We further assume $\phi_\pi = 1.5$. In terms of the shock, we assume that $r_L = -0.01$. This implies a natural rate of interest of

[18] According to NIPA data 36.3% of all exhaustive government expenditures in the US are determined at the federal level. In the EU there is a common budget. However, it is very small and consists mostly of transfers. There is basically no exhaustive government spending administrated at the area-wide level.

[19] The average import share in the euro area during 1999–2006 is closer to 50 percent, but this accounts for trade with countries outside of the euro area. The share of intra-euro area imports in GDP is about 17 percent according to the Monthly Foreign Trade Statistics of the OECD.

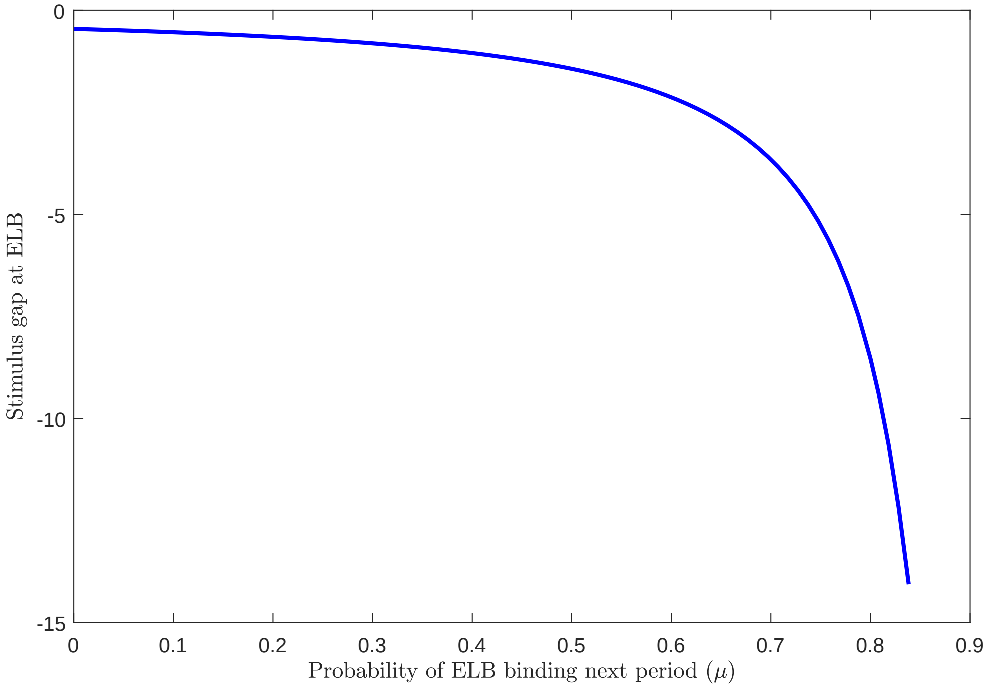

Figure 2: Fiscal stimulus gap at the ELB. Difference of optimal increase of government consumption without and with coordination, measured in percentage points along the vertical axis ($\hat{g}_L^{*,Nash} - \hat{g}_L^{*,Coord}$), horizontal axis measures probability that the ELB remains binding for another period (μ).

-4% (annualized) for the ELB scenario. For all experiments we verify that the ELB remains binding also as government spending is raised in response to the shock. Table 1 summarizes the parameter values.

Given these parameter values, we solve the model in the absence of coordination numerically as in Soderlind (1999). Appendix H provides details. As in our analysis above, we focus on the ELB scenario and, in particular, on the optimal adjustment of government spending. In the case of coordination, Proposition 1 provides an analytic solution. Figure 2 displays the fiscal stimulus gap at the ELB: the difference of the optimal increase of government spending without and with coordination. Specifically, the vertical axis measures the difference of the percentage change of government spending in percentage points ($\hat{g}_L^{*,Nash} - \hat{g}_L^{*,Coord}$). The horizontal axis measures the probability μ that the ELB remains binding for another period.[20] The figure illustrates that whether fiscal policies are coordinated or not hardly matters if the

[20]We restrict the range of μ to values for which we obtain a locally unique equilibrium.

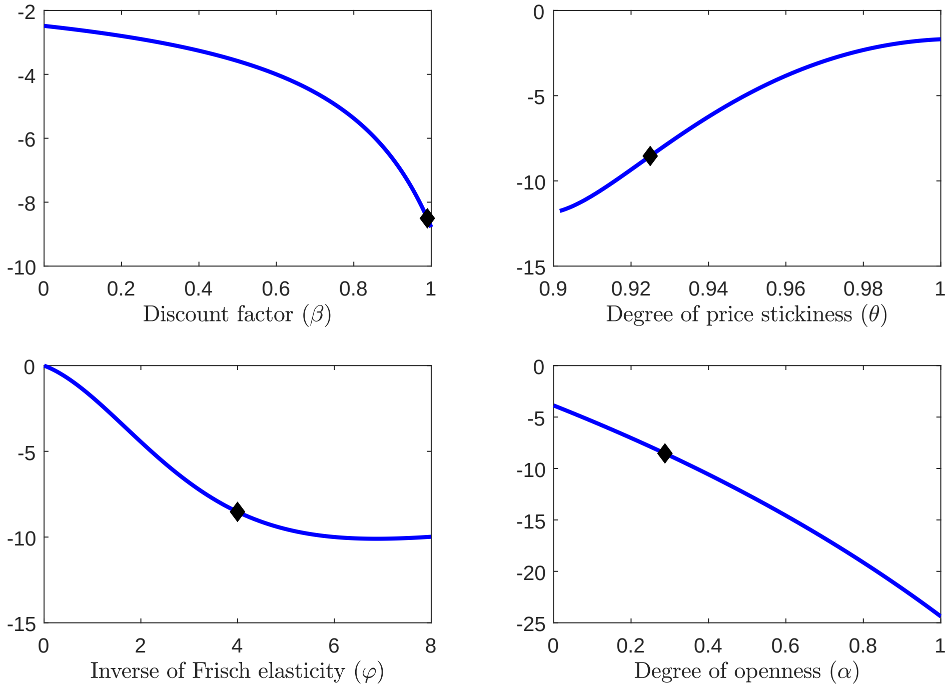

Figure 3: Stimulus gap conditional on the discount factor β (upper-left panel), the degree of price stickiness (upper-right panel), the inverse of the Frisch elasticity of labor supply φ (lower-left panel) and openness α (lower-right panel) for $\mu = 0.8$, see Figure 2. We only consider parameter values for which the equilibrium is locally unique. A diamond indicates parameter values under the baseline parameterization.

expected duration of the ELB episode is short. However, for larger values of μ the stimulus gap is sizable. If, for instance, the expected duration is five quarters ($\mu = 0.8$), the difference amounts to 8.5 percentage points.

We now illustrate to what extent this result depends on specific parameter values. For this purpose we vary one parameter at a time, while keeping μ fixed at 0.8 throughout. We first consider alternative values for the discount factor β. It determines how the dynamics of the terms of trade (as endogenous state variable) impact optimal policy in the absence of coordination (see the discussion in Section 3.2 above).[21] The upper-left panel of Figure 3 shows the stimulus gap as a function of β. We find that the stimulus gap increases as β increases or, put differently, the desire to avoid a persistent appreciation of the terms of trade increases with β in the absence of coordination.

[21] At the same time β matters also for the slope of the Phillips curve. Yet, as we vary the value of β, we keep κ constant (just like all other parameters) in order to focus on the role of the dynamic terms of trade effect.

The upper-right panel of Figure 3 shows the stimulus gap as a function of the degree of price stickiness θ. We find that the lower the degree of price stickiness, the larger the gap between the optimal policy under coordination and Nash. To understand this finding, note that inflation responds more strongly to higher government spending if prices are more flexible. Higher inflation at the union level reduces real interest rates and thus stimulates aggregate demand at the union level. At the country level, instead, higher inflation appreciates the terms of trade and thus reduces the demand for domestically produced goods. Hence, the more flexible prices, the more negative the stimulus gap.

The lower-left panel of Figure 3 shows the stimulus gap conditional on the inverse of the Frisch elasticity of labor supply φ. As the Frisch elasticity declines (that is, as φ increases), marginal costs and, hence, inflation respond more strongly to higher government spending. Put differently, the Phillips curve becomes steeper as φ increases, just like when θ declines, see equation (6). We therefore find the stimulus gap more negative, the larger the Frisch elasticity.

The lower-right panel of Figure 3 shows the stimulus gap conditional on openness parameter α. It is possible to show that, the higher the degree of openness, the stronger the impact of government spending on the terms of trade. Consequently, in a more open economy policy makers seek to avoid a stronger appreciation of the terms of trade in the midst of a severe recession and provide a lower stimulus in the absence of policy coordination. Note that the stimulus gap does not vanish in the closed economy limit ($\alpha = 0$) since monetary regimes still differ. While there is an implicit price-level target in place at the country-level, the price level features a unit root at the union level. This difference has a strong bearing on the transmission of fiscal policy (Corsetti et al., 2013b).

So far we have focused on the stimulus gap, that is for a given steady state we have computed the percentage point difference in the optimal variation of government spending with and without coordination. We have illustrated that the gap can be sizable. However, in Section 3.1 we have established that the *level* of government spending is too high to begin with in steady state in the absence of coordination. We therefore compute the optimal level of government spending in the ELB scenario to see which of these effects dominates and report results in Figure 4. The vertical axis measures the optimal level of government consumption with (dashed lines) and without (solid lines) coordination. Along the horizontal axis we measure again the expected duration of the ELB episode. Results are based on the parameter values listed in Table 1 above. In this case, we find that the steady-state effect dominates: the level of spending without coordination exceeds the optimal level with coordination for all

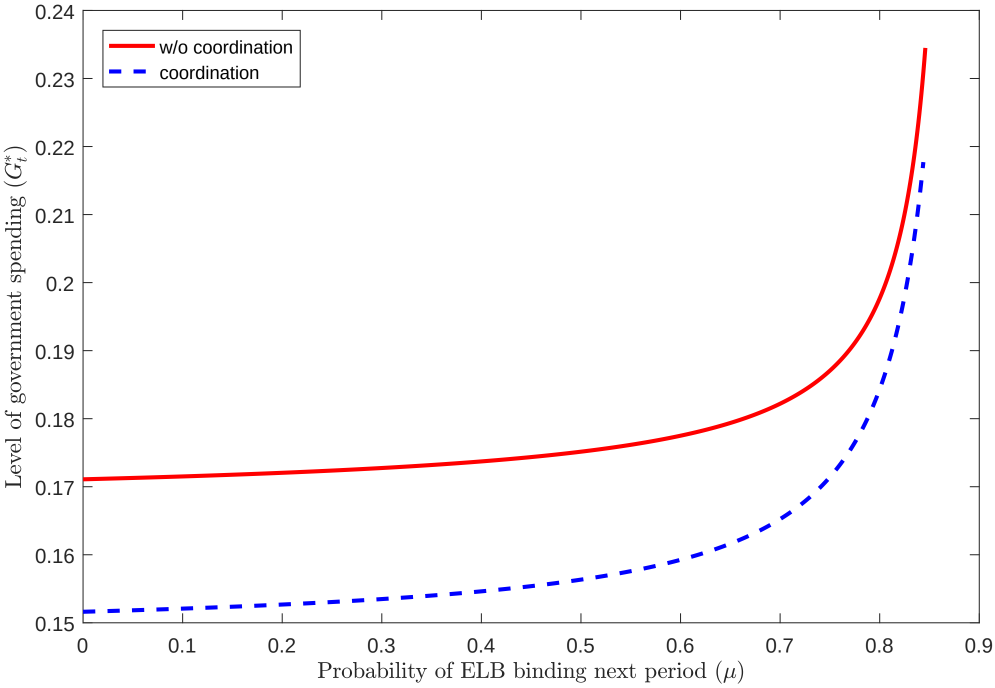

Figure 4: The optimal level of government spending under coordination (dashed lines) and without (solid lines) (i.e. G_t^{Coord} and G_t^{Nash}). We only compute results for combinations of parameters which give rise to a locally unique equilibrium.

values of μ for which we obtain a unique equilibrium.[22] Nevertheless, the distance between the optimal level of government expenditure with and without coordination becomes smaller. Against this background, one may conjecture that there is little need to coordinate on fiscal stimulus at the ELB. In order to assess this conjecture formally, we evaluate welfare under the optimal policy with and without coordination. Specifically, we compute the consumption equivalent ζ which compensates the household for the lack of coordination (see Appendix I for details). Figure 5 displays the result. The vertical axis measures the consumption equivalent in percentage points of steady-state consumption in the absence of coordination. The horizontal axis measures the probability μ that the ELB remains binding for another period. The longer the ELB is expected to bind, the higher the consumption equivalent. Hence, the need for coordination increases in the duration of the ELB—despite the fact that the distance between the optimal level of spending shrinks as μ increases (see Figure 4). This

[22] In principle, it is possible that the optimal level of spending without coordination falls short of the optimal level with coordination. For instance, assuming $\varphi = 0.2$ and $\theta = 0.5$ we find this to be the case, provided the ELB episode is expected to be sufficiently long lasting.

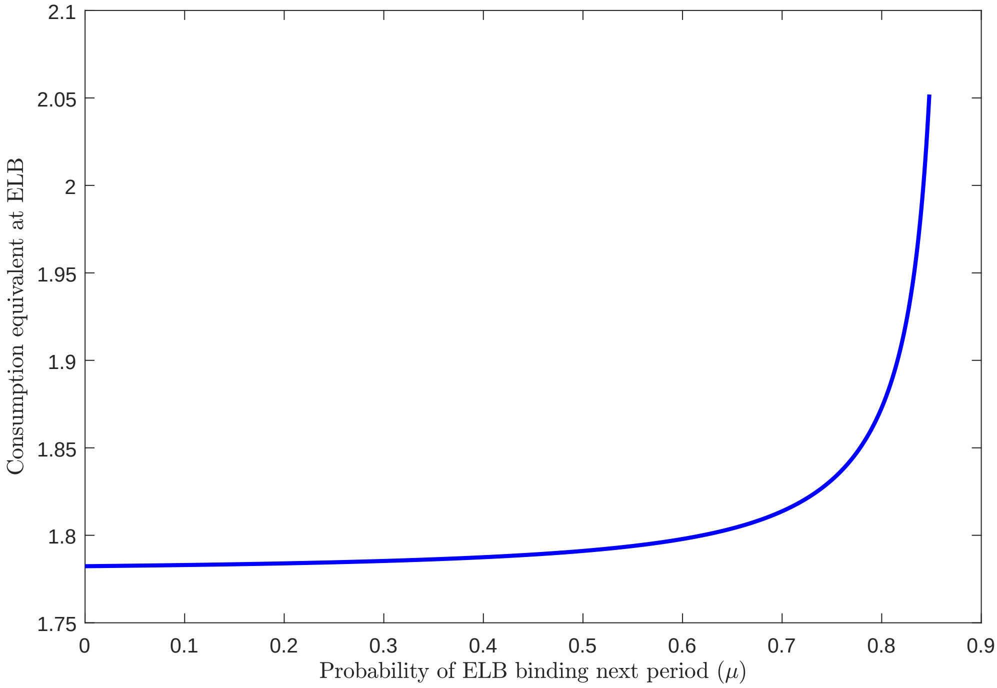

Figure 5: Welfare loss due to lack of coordination. Consumption equivalent which compensates for the lack of coordination expressed in percentage points of steady state consumption in the absence of coordination (see Appendix I).

is because the deviations from the optimal allocation become more costly, the further away the economy moves from the steady-state under coordination.

Finally, Figure 5 also shows that the largest part of the welfare loss is due to absence of coordination at steady state ($\zeta = 1.78$ for $\mu = 0$). This is perhaps not surprising since the economy will return permanently to steady state, once the ELB ceases to bind. Still, when the expected duration of the ELB is 5 quarters ($\mu = 0.8$) about 15% of the consumption equivalent are due to lack of coordination at the ELB. This strikes us as sizable, given the rather short duration of the ELB scenario.

5 Conclusion

In the context of the global financial crisis fiscal policy has been rediscovered as a stabilization tool. A central concern in this regard is that monetary policy may become constrained by the ELB on nominal interest rates precisely at times when the need for stabilization is particularly large. In this case it not only seems natural to turn to fiscal policy for additional support, it has also been documented that fiscal policy is likely to be particularly effective under such circumstances.

Against this background, we consider a currency union where a common monetary policy operates jointly with many fiscal policies. Assuming that the common monetary policy is unable to stabilize area-wide inflation and output, we ask whether there is a need to coordinate government spending policies across the member states of the union. This possibility arises because uncoordinated fiscal policies induce movements of the terms of trade which are internalized by a coordinated policy response.

Our analysis is based on the model of Galí and Monacelli (2008) which we extend in order to account for the ELB and the absence of fiscal policy coordination. Absent coordination, we find—in line with earlier results—that each country seeks to improve its terms of trade as long as the ELB is not binding. In a symmetric equilibrium, however, the terms of trade are unchanged and government spending is too high relative to the outcome under coordination. At the ELB, instead, we find that the optimal fiscal response implies too little fiscal stimulus in the absence of coordination. In this case governments seek to avoid an appreciation of the terms of trade in order to prevent a loss of competitiveness in times of economic slack.

Hence, absent coordination, the terms-of-trade externality has a differential impact on the optimal level of government spending depending on whether the ELB binds or not. Accounting for both dimensions, we find that the welfare loss due to the lack of coordination increases in the expected duration of the ELB episode. We thus conclude that there is indeed a case for coordinating fiscal stabilization policies in a currency union, once monetary policy is constrained by an ELB.

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A Steady state under coordination

Note that the risk sharing condition and the market-clearing condition in (15) imply

$$C^i = (Y^i - G^i)^{1-\alpha}(C^*)^\alpha. \tag{A.1}$$

The Lagrangian associated with problem (15) is given by

$$\mathcal{L} = \int_0^1 \left((1-\chi)\log C^i + \chi \log G^i - \frac{\left(N^i\right)^{1+\varphi}}{1+\varphi} \right) di$$
$$+ \int_0^1 \Lambda^i (C^i - (N^i - G^i)^{1-\alpha}(C^*)^\alpha) di - \mu \left(C^* - \int_0^1 C^i di \right).$$

First order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial C^i} = (1-\chi)\frac{1}{C^i} + \Lambda^i + \mu = 0 \tag{A.2}$$
$$\frac{\partial \mathcal{L}}{\partial C^*} = \Lambda^i \alpha (N^i - G^i)^{1-\alpha}(C^*)^{\alpha-1} - \mu = 0 \tag{A.3}$$
$$\frac{\partial \mathcal{L}}{\partial N^i} = -(N^i)^\varphi - \Lambda^i(1-\alpha)(N^i - G^i)^{-\alpha}(C^*)^\alpha$$
$$= -(N^i)^\varphi - \Lambda^i(1-\alpha)\frac{C^i}{Y^i - G^i} = 0 \tag{A.4}$$
$$\frac{\partial \mathcal{L}}{\partial G^i} = \chi\frac{1}{G^i} + \Lambda^i(1-\alpha)(N^i - G^i)^{-\alpha}(C^*)^\alpha$$
$$= \chi\frac{1}{G^i} + \Lambda^i(1-\alpha)\frac{C^i}{Y^i - G^i} = 0. \tag{A.5}$$

Combining (A.2) and (A.3) yields:

$$(1-\chi)\frac{1}{C^i} + \Lambda^i = -\Lambda^i \alpha (N^i - G^i)^{1-\alpha}(C^*)^{\alpha-1}. \tag{A.6}$$

Combining (A.4) and (A.5) gives:

$$(N^i)^\varphi = \chi\frac{1}{G^i}. \tag{A.7}$$

Further, combining (A.5) and (A.6) gives:

$$(1-\chi)\frac{1}{C^i} - \chi\frac{1}{G^i}\frac{1}{1-\alpha}\frac{Y^i - G^i}{C^i} = -\chi\frac{1}{G^i}\frac{1}{1-\alpha}\frac{Y^i - G^i}{C^i}\alpha(N^i - G^i)^{1-\alpha}(C^*)^{\alpha-1}. \tag{A.8}$$

One can guess and verify that the solution under coordination, that is, to equations (A.7) and (A.8) is given by (see Galí and Monacelli, 2008):

$$C^i = 1-\chi$$
$$C^* = 1-\chi$$
$$G^i = \chi$$
$$Y^i = N^i = 1.$$

Hence, the steady-state ratio of government spending to output in any given country $i \in [0,1]$ is given by:

$$\gamma^{Coord} \equiv \left(\frac{G}{Y}\right)^{Coord} = \chi,$$

while output is given by:

$$Y^{Coord} = 1.$$

B Steady state in the absence of coordination

B.1 Planner problem

The Lagrangian associated with problem (17) is given by

$$\mathcal{L} = (1-\chi)\log C^i + \chi \log G^i - \frac{(N^i)^{1+\varphi}}{1+\varphi} + \Lambda(C^i - (N^i - G^i)^{1-\alpha}(C^*)^\alpha).$$

First order conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial C^i} = (1-\chi)\frac{1}{C^i} + \Lambda = 0 \tag{B.9}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial N^i} &= -(N^i)^\varphi - \Lambda(1-\alpha)(N^i - G^i)^{-\alpha}(C^*)^\alpha \\ &\overset{(A.1)}{=} -(N^i)^\varphi - \Lambda(1-\alpha)\frac{C^i}{Y^i - G^i} = 0\end{aligned} \tag{B.10}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial G^i} &= \chi\frac{1}{G^i} + \Lambda(1-\alpha)(N^i - G^i)^{-\alpha}(C^*)^\alpha \\ &= \chi\frac{1}{G^i} + \Lambda(1-\alpha)\frac{C^i}{Y^i - G^i} = 0.\end{aligned} \tag{B.11}$$

Combine (B.10) and (B.11) to get:

$$(N^i)^\varphi = \chi\frac{1}{G^i}. \tag{B.12}$$

Further, combine (B.9) and (B.11):

$$\frac{\chi}{1-\chi}\frac{C^i}{G^i} = (1-\alpha)\frac{C^i}{Y^i - G^i}.$$

Which can be rearranged to:

$$G^i = \frac{\chi}{1-\chi}\left((1-\alpha) + \frac{\chi}{1-\chi}\right)^{-1} Y^i. \tag{B.13}$$

It thus follows for the steady state in the absence of coordination (Nash) in any given country $i \in [0,1]$ that:

$$\left(\frac{G}{Y}\right)^{Nash} = \frac{\chi}{(1-\alpha)(1-\chi)+\chi}. \tag{B.14}$$

Since in a symmetric steady state $Y^i = N^i$, combining (B.12) and (B.14) gives:

$$N^{Nash} = Y^{Nash} = [(1-\alpha)(1-\chi)+\chi]^{\frac{1}{1+\varphi}} \,. \tag{B.15}$$

Further, it holds that $G^{Coord} < G^{Nash}$ since

$$G^{Coord} = \chi < \frac{\chi}{(1-\alpha)(1-\chi)+\chi}\,[(1-\alpha)(1-\chi)+\chi]^{\frac{1}{1+\varphi}} = G^{Nash}.$$

B.2 Decentralization of the planner solution in steady state

In the following we show how the planner allocation in the absence of coordination can be decentralized in a symmetric zero-inflation steady state. Unless offset by the employment subsidy, firms choose a constant mark-up over marginal costs MC^i, which can be expressed as (see equation (41) in Galí and Monacelli, 2008):

$$1 - \frac{1}{\varepsilon} = MC^i = \frac{1-\tau^i}{1-\chi}(N^i)^{1+\varphi}\left(1 - \frac{G^i}{Y^i}\right).$$

In order to decentralize the planner solution government consumption, G^i, has to follow a rule according to (B.13). Solving the resulting expression for N^i gives:

$$(N^i)^{1+\varphi} = \left(1-\frac{1}{\varepsilon}\right)\frac{1-\chi}{1-\tau^i}\left(1 - \frac{\chi}{1-\chi}\left((1-\alpha)+\frac{\chi}{1-\chi}\right)^{-1}\right)^{-1}.$$

By additionally choosing the following subsidy

$$1-\tau^i = \left(1-\frac{1}{\varepsilon}\right)(1-\alpha)^{-1}, \tag{B.16}$$

the social planner solution without coordination is decentralized in a symmetric zero-inflation steady state.

C Deriving the welfare function without coordination

In the absence of coordination there are linear terms in a second-order approximation to household utility. We follow the approach of Benigno and Woodford (2006) and substitute for the linear terms using a second order approximation to the market-clearing condition. In the following we drop the country index i for simplicity and approximate the percentage deviation of a generic variable X_t from its steady state X by

$$\frac{X_t - X}{X} \approx \hat{x}_t + \frac{1}{2}\hat{x}_t^2,$$

where $\hat{x}_t = x_t - x$ and $x_t = \log X_t$.

C.1 Second order approximation to the goods market clearing condition

The market-clearing condition is given by $Y_t = C_t S_t^{\alpha} + G_t$. Taking logs and rearranging gives:

$$\log C_t = \log(Y_t - G_t) - \alpha \log S_t.$$

Using the second-order approximation for $\log(Y_t - G_t)$ around a generic steady state from the appendix in Galí and Monacelli (2008) and noting that a second-order approximation of $\log C_t$ and $\log S_t$ is in fact linear we obtain:

$$\hat{c}_t \approx \frac{1}{1-\gamma}\left(\hat{y}_t - \gamma \hat{g}_t\right) - \frac{1}{2}\frac{\gamma}{(1-\gamma)^2}(\hat{g}_t - \hat{y}_t)^2 - \alpha s_t. \tag{C.17}$$

Combining the above equation with the risk-sharing condition (3) yields:

$$0 \approx \frac{1}{1-\gamma}\left(\hat{y}_t - \gamma \hat{g}_t\right) - \frac{1}{2}\frac{\gamma}{(1-\gamma)^2}(\hat{g}_t - \hat{y}_t)^2 - s_t + t.i.p. \tag{C.18}$$

where $(t.i.p.)$ captures terms independent of policy. This includes $\hat{c}_t^*$ since it evolves exogenously for a given member of the currency union. For future reference, we define:

$$A_y \equiv \frac{1}{1-\gamma}; \quad A_g \equiv -\frac{\gamma}{1-\gamma}; \quad A_s \equiv -1. \tag{C.19}$$

C.2 Second order approximation to utility

Utility $U_t = U(C_t, G_t, N_t)$ is additively separable in its arguments. A second order approximation around a generic steady state C, G, N therefore gives:

$$\begin{aligned} U_t - U \approx\ & U_C C\left(\frac{C_t - C}{C}\right) + U_G G\left(\frac{G_t - G}{G}\right) + U_N N\left(\frac{N_t - N}{N}\right) \\ & + \frac{1}{2}U_{CC}C^2\left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2}U_{GG}G^2\left(\frac{G_t - G}{G}\right)^2 \\ & + \frac{1}{2}U_{NN}N^2\left(\frac{N_t - N}{N}\right)^2. \end{aligned}$$

Rewriting the expression in terms of log deviations the above approximation becomes:

$$\begin{aligned} U_t - U \approx\ & U_C C\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + U_G G\left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2\right) + U_N N\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2\right) \\ & + \frac{1}{2}U_{CC}C^2\hat{c}_t^2 + \frac{1}{2}U_{GG}G^2\hat{g}_t^2 + \frac{1}{2}U_{NN}N^2\hat{n}_t^2. \end{aligned}$$

Rearranging:

$$\begin{aligned} U_t - U \approx\ & U_C C\left(\hat{c}_t + \frac{1}{2}\left(1 + \frac{U_{CC}C}{U_C}\right)\hat{c}_t^2\right) + U_G G\left(\hat{g}_t + \frac{1}{2}\left(1 + \frac{U_{GG}G}{U_G}\right)\hat{g}_t^2\right) \\ & + U_N N\left(\hat{n}_t + \frac{1}{2}\left(1 + \frac{U_{NN}N}{U_N}\right)\hat{n}_t^2\right). \end{aligned}$$

Defining further: $\sigma \equiv -\frac{U_{CC}C}{U_C}$, $\sigma_g \equiv -\frac{U_{GG}G}{U_G}$ and $\sigma_n \equiv \frac{U_{NN}N}{U_N}$ yields

$$\begin{aligned} U_t - U \approx\ & U_C C\left(\hat{c}_t + \frac{1}{2}\left(1-\sigma\right)\hat{c}_t^2\right) + U_G G\left(\hat{g}_t + \frac{1}{2}\left(1-\sigma_g\right)\hat{g}_t^2\right) \\ & + U_N N\left(\hat{n}_t + \frac{1}{2}\left(1+\sigma_n\right)\hat{n}_t^2\right). \end{aligned}$$

Since utility is given by

$$U_t = (1-\chi)\log C_t + \chi \log G_t - \frac{N_t^{1+\varphi}}{1+\varphi},$$

the above defined parameters become: $\sigma = \sigma_g = 1$ while $\sigma_n = \varphi$, such that we get:

$$\frac{U_t - U}{U_C C} \approx\ \hat{c}_t + \frac{U_G G}{U_C C}\hat{g}_t + \frac{U_N N}{U_C C}\left(\hat{n}_t + \frac{1}{2}(1+\varphi)\hat{n}_t^2\right).$$

Because of monopolistic competition firms charge a markup over marginal costs. If not offset by a certain value for the labor subsidy there will be a wedge Φ between the marginal rate of substitution and the marginal product of labor (MPN) in steady state (see, for instance, Galí, 2008, p.106):

$$-\frac{U_N}{U_C} = \text{MPN}(1-\Phi). \tag{C.20}$$

In our setup we have $\text{MPN} = Y/N$. Therefore

$$\frac{U_N}{U_C}\frac{N}{C} = -\frac{1}{1-\gamma}(1-\Phi),$$

with $1-\gamma = C/Y$. Making use of the above expression and the one for $\frac{U_G G}{U_C C}$ under the assumed utility function, we can rewrite the approximation to utility as:

$$\frac{U_t - U}{U_C C} \approx\ \hat{c}_t + \frac{\chi}{1-\chi}\hat{g}_t - \frac{1-\Phi}{1-\gamma}\left(\hat{n}_t + \frac{1}{2}(1+\varphi)\hat{n}_t^2\right).$$

Further, we use equation (C.17) in order to substitute for $\hat{c}_t$. Therefore

$$\begin{aligned} \frac{U_t - U}{U_C C} \approx\ & \frac{1}{1-\gamma}\left(\hat{y}_t - \gamma\hat{g}_t\right) - \frac{1}{2}\frac{\gamma}{(1-\gamma)^2}(\hat{g}_t - \hat{y}_t)^2 - \alpha s_t \\ & + \frac{\chi}{1-\chi}\hat{g}_t - \frac{1-\Phi}{1-\gamma}\left(\hat{n}_t + \frac{1}{2}(1+\varphi)\hat{n}_t^2\right). \end{aligned}$$

In order to substitute for $\hat{n}_t$, it can be shown that aggregate labor demand is given by $N_t = Y_t \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj$, see Galí and Monacelli (2008). Define $z_t \equiv \log \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj$. Thus, it holds around a symmetric steady state that:

$$\hat{n}_t = \hat{y}_t + z_t.$$

Further it can be shown that z_t is of second order with $z_t \approx \frac{1}{2}\frac{\varepsilon}{\lambda}\pi_t^2$, see again Galí and Monacelli (2008). Finally, the approximation to utility can be expressed as:

$$\begin{aligned}\frac{U_t - U}{U_C C} \approx\; & \frac{\Phi}{1-\gamma}\hat{y}_t + \left(\frac{\chi}{1-\chi} - \frac{\gamma}{1-\gamma}\right)\hat{g}_t - \alpha s_t \\ & - \frac{1}{2}\frac{\gamma}{(1-\gamma)^2}(\hat{g}_t - \hat{y}_t)^2 - \frac{1}{2}\frac{1-\Phi}{1-\gamma}(1+\varphi)\hat{y}_t^2 - \frac{1}{2}\frac{1-\Phi}{1-\gamma}\frac{\varepsilon}{\lambda}\pi_t^2. \end{aligned} \tag{C.21}$$

Define:

$$B_y \equiv \frac{\Phi}{1-\gamma}; \quad B_g \equiv \frac{\chi}{1-\chi} - \frac{\gamma}{1-\gamma}; \quad B_s \equiv -\alpha. \tag{C.22}$$

Note that the welfare function under coordination can be obtained from (C.21) if one approximates around the efficient steady state from the union wide perspective. In that steady state the labor subsidy is chosen such that $\Phi = 0$ and γ equals χ since government spending is chosen efficiently. The terms of trade drop out by computing union-wide welfare, that is by taking the integral of (C.21) over all $i \in [0, 1]$.

C.3 The welfare function—substituting for the linear terms

Absent coordination, government spending and the employment subsidy, τ, are not chosen efficiently such that $\gamma = \gamma^{Nash} \neq \chi$ and $\Phi \neq 0$. Specifically, in a symmetric steady state the distortion Φ is given by (see Galí, 2008, p.73 and p.106):

$$\Phi = 1 - \frac{1}{\frac{\varepsilon}{\varepsilon-1}(1-\tau)}.$$

Inserting for the subsidy according to (B.16) yields:

$$\Phi = \alpha.$$

By inserting for γ^{Nash} and Φ in (C.19) and (C.22) we get:

$$\begin{aligned} A_y &= \frac{(1-\chi)(1-\alpha)+\chi}{(1-\chi)(1-\alpha)}; \quad A_g = -\frac{\chi}{(1-\chi)(1-\alpha)}; \quad A_s = -1; \\ B_y &= \frac{\alpha[(1-\chi)(1-\alpha)+\chi]}{(1-\chi)(1-\alpha)}; \quad B_g = -\frac{\alpha\chi}{(1-\chi)(1-\alpha)}; \quad B_s = -\alpha. \end{aligned}$$

Thus, it is easily seen that subtracting α times condition (C.18) from (C.21)—both evaluated at the Nash steady state—removes the linear terms from the approximation to utility. As a result, the welfare function is given by:

$$\mathbb{W}_t^{Nash} \approx -\frac{1}{2}\left(\frac{\varepsilon}{\lambda}(\pi_t^i)^2 + (1+\varphi)(\hat{y}_t^i)^2 + \frac{\gamma^{Nash}}{1-\gamma^{Nash}}\left(\hat{g}_t^i - \hat{y}_t^i\right)^2\right) + t.i.p.$$

D Optimal policy with coordination

The Lagrangian associated with problem (19) is given by

$$
\begin{aligned}
\mathcal{L}_t = &-\frac{1}{2}\left(\frac{\varepsilon}{\lambda}(\pi_t^*)^2 + (1+\varphi)(\hat{y}_t^*)^2 + \frac{\gamma^{Coord}}{1-\gamma^{Coord}}(\hat{g}_t^* - \hat{y}_t^*)^2\right) \\
&+ \xi_{0,t}^*\left[\pi_t^* - \lambda\left(\frac{1}{1-\gamma^{Coord}} + \varphi\right)\hat{y}_t^* + \frac{\lambda\gamma^{Coord}}{1-\gamma^{Coord}}\hat{g}_t^* - \nu_{0,t}^*\right] \\
&+ \xi_{1,t}^*\left[-(\hat{y}_t^* - \gamma\hat{g}_t^*) + \nu_{1,t}^*\right].
\end{aligned}
$$

The Kuhn-Tucker conditions read as follows:

$$\frac{\partial \mathcal{L}_t}{\partial \pi_t^*} = -\frac{\varepsilon}{\lambda}\pi_t^* + \xi_{0,t}^* = 0 \tag{D.23}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{y}_t^*} = -(1+\varphi)\hat{y}_t^* + \frac{\gamma^{Coord}}{1-\gamma^{Coord}}(\hat{g}_t^* - \hat{y}_t^*) - \lambda\left(\frac{1}{1-\gamma^{Coord}} + \varphi\right)\xi_{0,t}^* - \xi_{1,t}^* = 0 \tag{D.24}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{g}_t^*} = -\frac{\gamma^{Coord}}{1-\gamma^{Coord}}(\hat{g}_t^* - \hat{y}_t^*) + \frac{\lambda\gamma^{Coord}}{1-\gamma^{Coord}}\xi_{0,t}^* + \gamma^{Coord}\xi_{1,t}^* = 0 \tag{D.25}$$

$$\xi_{1,t}^*(-(\hat{y}_t^* - \gamma\hat{g}_t^*) + \nu_{1,t}^*) = 0$$

$$\xi_{1,t}^* \geq 0; \;\; -(\hat{y}_t^* - \gamma\hat{g}_t^*) + \nu_{1,t}^* \geq 0.$$

Case 1 The effective-lower-bound constraint does not bind: $\xi_{1,t}^* = 0$ and $-(\hat{y}_t^* - \gamma\hat{g}_t^*) + \nu_{1,t}^* \geq 0$. Equation (D.25) thus implies for $\xi_{0,t}^*$

$$\xi_{0,t}^* = \frac{1}{\lambda}(\hat{g}_t^* - \hat{y}_t^*). \tag{D.26}$$

Inserting for $\xi_{0,t}^*$ in (D.24) yields

$$
\begin{aligned}
0 &= -(1+\varphi)\hat{y}_t^* + \frac{\gamma^{Coord}}{1-\gamma^{Coord}}(\hat{g}_t^* - \hat{y}_t^*) - \lambda\left(\frac{1}{1-\gamma^{Coord}} + \varphi\right)\frac{1}{\lambda}(\hat{g}_t^* - \hat{y}_t^*) \\
0 &= -(1+\varphi)\hat{y}_t^* - (1+\varphi)(\hat{g}_t^* - \hat{y}_t^*) \\
\hat{g}_t^* &= 0.
\end{aligned}
$$

Thus by (D.26)

$$\xi_{0,t}^* = -\frac{1}{\lambda}\hat{y}_t^*. \tag{D.27}$$

Combining (D.27) and (D.23):

$$\hat{y}_t^* = -\varepsilon\pi_t^*.$$

Therefore, when it is optimal to stabilize output at steady state inflation should be zero and vice versa. And indeed, considering the functional form of the welfare function it is clear that $\pi_t^* = \hat{y}_t^* = \hat{g}_t^* = 0$ is the global maximum.

Case 2 The effective-lower-bound constraint binds: $\xi^*_{1,t} > 0$ and $-(\hat{y}^*_t - \gamma\hat{g}^*_t) + \nu^*_{1,t} = 0$. Rearrange (D.25):

$$-\xi^*_{1,t} = -\frac{1}{1-\gamma^{Coord}}(\hat{g}^*_t - \hat{y}^*_t) + \frac{\lambda}{1-\gamma^{Coord}}\xi^*_{0,t}. \tag{D.28}$$

Combining it with (D.24):

$$\begin{aligned}
0 &= -(1+\varphi)\hat{y}^*_t + \frac{\gamma^{Coord}}{1-\gamma^{Coord}}(\hat{g}^*_t - \hat{y}^*_t) - \lambda\left(\frac{1}{1-\gamma^{Coord}} + \varphi\right)\xi^*_{0,t} \\
&\quad - \frac{1}{1-\gamma^{Coord}}(\hat{g}^*_t - \hat{y}^*_t) + \frac{\lambda}{1-\gamma^{Coord}}\xi^*_{0,t} \\
\xi^*_{0,t} &= -\frac{1}{\lambda}\hat{y}^*_t - \frac{1}{\lambda\varphi}\hat{g}^*_t. \qquad (D.29)
\end{aligned}$$

Inserting for $\xi^*_{0,t}$ in (D.23) yields after rearranging

$$\pi^*_t + \frac{1}{\varepsilon}\hat{y}^*_t = -\psi^{Coord}_g \hat{g}^*_t, \tag{D.30}$$

where $\psi^{Coord}_g \equiv \frac{1}{\varepsilon\varphi}$.

E Optimal policy in the absence of coordination

The Lagrangian associated with problem (21) is given by

$$\begin{aligned}
\mathcal{L}_t = &-\frac{1}{2}\left(\frac{\varepsilon}{\lambda}(\pi^i_t)^2 + (1+\varphi)(\hat{y}^i_t)^2 + \frac{\gamma}{1-\gamma}\left(\hat{g}^i_t - \hat{y}^i_t\right)^2\right) + \beta E_t V(s^i_t, \pi^*_{t+1}, \hat{c}^*_{t+1}) \\
&+ m_{1,t}\left[\hat{y}^i_t - \gamma\hat{g}^i_t - (1-\gamma)\hat{c}^*_t - (1-\gamma)s^i_t\right] \\
&+ m_{2,t}\left[\pi^i_t - \beta E_t\pi^i_{t+1} - \lambda\left(\frac{1}{1-\gamma} + \varphi\right)\hat{y}^i_t + \lambda\frac{\gamma}{1-\gamma}\hat{g}^i_t\right] \\
&+ m_{3,t}\left[\pi^i_t - \pi^*_t + s^i_t - s^i_{t-1}\right] \\
&\text{and } E_t\pi^i_{t+1} \text{ given.}
\end{aligned}$$

Note that while $E_t\pi^i_{t+1}$ is taken as given, it is a given function of today's state variable s_t. First order conditions

$$\frac{\partial\mathcal{L}_t}{\partial\pi^i_t} = -\frac{\varepsilon}{\lambda}\pi^i_t + m_{2,t} + m_{3,t} = 0 \tag{E.31}$$

$$\frac{\partial\mathcal{L}_t}{\partial\hat{y}^i_t} = -(1+\varphi)\hat{y}^i_t + \frac{\gamma}{1-\gamma}(\hat{g}^i_t - \hat{y}^i_t) + m_{1,t} - \lambda\left(\frac{1}{1-\gamma} + \varphi\right)m_{2,t} = 0 \tag{E.32}$$

$$\frac{\partial\mathcal{L}_t}{\partial\hat{g}^i_t} = -\frac{\gamma}{1-\gamma}(\hat{g}^i_t - \hat{y}^i_t) - \gamma m_{1,t} + \frac{\lambda\gamma}{1-\gamma}m_{2,t} = 0 \tag{E.33}$$

$$\frac{\partial\mathcal{L}_t}{\partial s^i_t} = \beta E_t\frac{\partial V}{\partial s^i_t} - (1-\gamma)m_{1,t} - m_{2,t}\frac{\beta\partial E_t\pi^i_{t+1}}{\partial s^i_t} + m_{3,t} = 0. \tag{E.34}$$

Solving first order conditions (E.32) and (E.33) for $m_{2,t}$ yields

$$m_{2,t} = -\frac{1}{\lambda}\hat{y}_t^i - \frac{1}{\lambda\varphi}\hat{g}_t^i.$$

This implies for $m_{3,t}$ by (E.31)

$$m_{3t} = \frac{\varepsilon}{\lambda}\pi_t^i + \frac{1}{\lambda}\hat{y}_t^i + \frac{1}{\lambda\varphi}\hat{g}_t^i.$$

First order condition (E.33) thus requires for $m_{1,t}$ that

$$m_{1,t} = -\frac{1}{1-\gamma}(\hat{g}_t^i - \hat{y}_t^i) - \frac{1}{1-\gamma}\hat{y}_t^i - \frac{1}{1-\gamma}\frac{1}{\varphi}\hat{g}_t^i.$$

Finally, substituting for the multipliers in (E.34) gives

$$-\lambda\beta E_t\frac{\partial V}{\partial s_t^i} + \beta\left(\hat{y}_t^i + \frac{1}{\varphi}\hat{g}_t^i\right)\frac{\partial E_t\pi_{t+1}^i}{\partial s_t} + \frac{1}{\varepsilon}\hat{y}_t^i + \pi_t^i = -\psi_g^{Nash}\hat{g}_t^i,$$

with $\psi_g^{Nash} \equiv \frac{1}{\varepsilon\varphi}\left(\lambda\varphi + (1+\lambda)\right)$. Taking the limit of $\beta \to 0$ the above condition becomes

$$\frac{1}{\varepsilon}\hat{y}_t^i + \pi_t^i = -\psi_g^{Nash}\hat{g}_t^i. \tag{E.35}$$

F Proof of Proposition 1

In Proposition 1 we state the solution for optimal government consumption at the ZLB with coordination. We prove the proposition more generally in order to use results for the proof of proposition 2. That is, we index regime dependent parameters by $j \in \{Coord, Nash\}$. Optimal policy at the ELB is determined by equations (12), (14) and (D.30) or (E.35) respectively. The equations are repeated here for convenience:

$$\pi_L^{*,j} = \frac{1}{1-\beta\mu}\kappa^j(\hat{y}_L^{*,j} - \frac{\bar{\sigma}^j\gamma^j}{\bar{\sigma}^j+\varphi}\hat{g}_L^{*,j}) \tag{F.36}$$

$$\hat{y}_L^{*,j} = \frac{(1-\gamma^j)(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-(1-\gamma^j)\mu\kappa^j}r_L + \frac{(1-\mu)(1-\beta\mu)\gamma^j - (1-\gamma^j)\mu\kappa^j\frac{\gamma^j\bar{\sigma}^j}{\bar{\sigma}^j+\varphi}}{(1-\mu)(1-\beta\mu)-(1-\gamma^j)\mu\kappa^j}\hat{g}_L^{*,j} \tag{F.37}$$

$$0 = \pi_L^{*,j} + \frac{1}{\varepsilon}\hat{y}_L^{*,j} + \psi_g^j\,\hat{g}_L^{*,j}. \tag{F.38}$$

Equations (F.36) and (F.38) imply:

$$-\frac{1}{\varepsilon}\hat{y}_L^{*,j} - \psi_g^j\hat{g}_L^{*,j} = \frac{1}{1-\beta\mu}\kappa^j(\hat{y}_L^{*,j} - \frac{\bar{\sigma}^j\gamma^j}{\bar{\sigma}^j+\varphi}\hat{g}_L^{*,j}).$$

Making use of $\frac{\kappa^j\bar{\sigma}^j\gamma^j}{\bar{\sigma}^j+\varphi} = \frac{\lambda\gamma^j}{1-\gamma^j}$, the above equation can be rearranged to:

$$\hat{y}_L^{*,j} = \frac{\frac{\varepsilon\lambda\gamma^j}{1-\gamma^j} - (1-\beta\mu)\varepsilon\psi_g^j}{\kappa^j\varepsilon + (1-\beta\mu)}\hat{g}_L^{*,j}. \tag{F.39}$$

Combining (F.37) and (F.39), making use of $\frac{\kappa^j \bar{\sigma}^j \gamma^j}{\bar{\sigma}^j+\varphi} = \frac{\lambda\gamma^j}{1-\gamma^j}$ again, yields:

$$\frac{\frac{\varepsilon\lambda\gamma^j}{1-\gamma^j} - (1-\beta\mu)\varepsilon\psi_g^j}{\kappa^j\varepsilon + (1-\beta\mu)}\hat{g}_L^{*,j} = \frac{(1-\gamma^j)(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - (1-\gamma^j)\mu\kappa^j} r_L + \frac{(1-\mu)(1-\beta\mu)\gamma^j - \mu\lambda\gamma^j}{(1-\mu)(1-\beta\mu) - (1-\gamma^j)\mu\kappa^j}\hat{g}_L^{*,j}.$$

Solving for $\hat{g}_L^{*,j}$ we get:

$$\hat{g}_L^{*,j} = -\Theta^j r_L,$$

with

$$\Theta^j = -\frac{(1-\gamma^j)(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - (1-\gamma^j)\mu\kappa^j}\left(\frac{\frac{\lambda\gamma^j\varepsilon}{1-\gamma^j} - (1-\beta\mu)\psi_g^j\varepsilon}{\kappa^j\varepsilon + (1-\beta\mu)} - \frac{(1-\mu)(1-\beta\mu)\gamma^j - \mu\lambda\gamma^j}{(1-\mu)(1-\beta\mu) - (1-\gamma^j)\mu\kappa^j}\right)^{-1}.$$

After rearranging, we obtain

$$\Theta^j = \frac{(1-\gamma^j)\left(\kappa^j\varepsilon + (1-\beta\mu)\right)}{\psi_g^j\varepsilon\left[(1-\mu)(1-\beta\mu) - (1-\gamma^j)\mu\kappa^j\right] + (1-\mu)\gamma^j\lambda\varphi\varepsilon + \gamma^j\left[(1-\mu)(1-\beta\mu) - \mu\lambda\right]}, \tag{F.40}$$

which is the expression in the main text. Since we consider only uniquely determined equilibria (see the ELB scenario in Section 2.2), the first term in the denominator is positive. Further, because $\lambda < (1-\gamma^j)\kappa^j$, also $(1-\mu)(1-\beta\mu) - \mu\lambda > 0$. All other expressions in Θ^j are non-negative. Hence, $\Theta^j > 0$. ■

G Proof of Proposition 2

For $\beta \to 0$ we obtain from (F.40)

$$\Theta^j = \frac{(1-\gamma^j)\left(\kappa^j\varepsilon + 1\right)}{\psi_g^j\varepsilon\left[(1-\mu) - (1-\gamma^j)\mu\kappa^j\right] + (1-\mu)\gamma^j\lambda\varphi\varepsilon + \gamma^j\left[(1-\mu) - \mu\lambda\right]}$$

Proposition 2 states that for $\beta \to 0$ it holds that $\hat{g}_L^{*,Nash} < \hat{g}_L^{*,Coord}$. Or put differently that $\Theta^{Nash} < \Theta^{Coord}$. Formally, we have:

$$\begin{aligned}&\frac{(1-\gamma^{Nash})\left(\kappa^{Nash}\varepsilon + 1\right)}{\psi_g^{Nash}\varepsilon\left[(1-\mu) - (1-\gamma^{Nash})\mu\kappa^{Nash}\right] + (1-\mu)\gamma^{Nash}\lambda\varphi\varepsilon + \gamma^{Nash}\left[(1-\mu) - \mu\lambda\right]}\\ <&\frac{(1-\gamma^{Coord})\left(\kappa^{Coord}\varepsilon + 1\right)}{\psi_g^{Coord}\varepsilon\left[(1-\mu) - (1-\gamma^{Coord})\mu\kappa^{Coord}\right] + (1-\mu)\gamma^{Coord}\lambda\varphi\varepsilon + \gamma^{Coord}\left[(1-\mu) - \mu\lambda\right]}.\end{aligned}$$

Since we showed in Appendix F that all terms in Θ^{Nash} and Θ^{Coord} are non-negative, we prove the above inequality by showing that the following holds:

$$\frac{\psi_g^{Coord}\varepsilon\left[(1-\mu) - (1-\gamma^{Coord})\mu\kappa^{Coord}\right] + (1-\mu)\gamma^{Coord}\lambda\varphi\varepsilon + \gamma^{Coord}\left[(1-\mu) - \mu\lambda\right]}{\psi_g^{Nash}\varepsilon\left[(1-\mu) - (1-\gamma^{Nash})\mu\kappa^{Nash}\right] + (1-\mu)\gamma^{Nash}\lambda\varphi\varepsilon + \gamma^{Nash}\left[(1-\mu) - \mu\lambda\right]} < 1 < \frac{(1-\gamma^{Coord})\left(\kappa^{Coord}\varepsilon + 1\right)}{(1-\gamma^{Nash})\left(\kappa^{Nash}\varepsilon + 1\right)}. \tag{G.41}$$

We start with the left hand side of (G.41):

$$\frac{\psi_g^{Coord}\varepsilon\left[(1-\mu)-(1-\gamma^{Coord})\mu\kappa^{Coord}\right]+(1-\mu)\gamma^{Coord}\lambda\varphi\varepsilon+\gamma^{Coord}\left[(1-\mu)-\mu\lambda\right]}{\psi_g^{Nash}\varepsilon\left[(1-\mu)-(1-\gamma^{Nash})\mu\kappa^{Nash}\right]+(1-\mu)\gamma^{Nash}\lambda\varphi\varepsilon+\gamma^{Nash}\left[(1-\mu)-\mu\lambda\right]}<1,$$

which can be rearranged to:

$$\begin{aligned}0<&(\psi_g^{Nash}-\psi_g^{Coord})(1-\mu)\varepsilon\\&-\psi_g^{Nash}(1-\gamma^{Nash})\varepsilon\mu\kappa^{Nash}+\psi_g^{Coord}(1-\gamma^{Coord})\varepsilon\mu\kappa^{Coord}\\&+(\gamma^{Nash}-\gamma^{Coord})(1-\mu)\lambda\varphi\varepsilon+(\gamma^{Nash}-\gamma^{Coord})\left[(1-\mu)-\mu\lambda\right].\end{aligned}$$

Inserting for ψ^j and κ^j with $j\in\{Coord, Nash\}$ yields after rearranging:

$$\begin{aligned}0<&\frac{1}{\varphi}\lambda(1+\varphi)(1-\mu)\\&-\frac{1}{\varphi}(1+\lambda(1+\varphi))\mu\lambda\left(1+(1-\gamma^{Nash})\varphi\right)+\frac{1}{\varphi}\mu\lambda\left(1+(1-\gamma^{Coord})\varphi\right)\\&+(\gamma^{Nash}-\gamma^{Coord})(1-\mu)\lambda\varphi\varepsilon+(\gamma^{Nash}-\gamma^{Coord})\left[(1-\mu)-\mu\lambda\right].\end{aligned}$$

This inequality can be further rearranged to:

$$0<\frac{1}{\varphi}\lambda(1+\varphi)\underbrace{\left((1-\mu)-(1-\gamma^{Nash})\mu\kappa^{Nash}\right)}_{>0}+\underbrace{(\gamma^{Nash}-\gamma^{Coord})}_{>0}(1-\mu)(\lambda\varphi\varepsilon+1),$$

where the sign of the highlighted terms follows from the condition on the uniqueness of equilibrium (see also Appendix F) and the relationship between the steady states with and without coordination. Hence, we have established that the left hand side is below unity.
We continue with the right hand side of (G.41) which— making use of the definition of κ—can be rearranged to:

$$(1-\gamma^{Coord})\left(\lambda(\frac{1}{1-\gamma^{Coord}}+\varphi)\varepsilon+1\right)>(1-\gamma^{Nash})\left(\lambda(\frac{1}{1-\gamma^{Nash}}+\varphi)\varepsilon+1\right).$$

The above expression can be further rearranged to:

$$(\gamma^{Nash}-\gamma^{Coord})(\lambda\varphi\varepsilon+1)>0,$$

which holds true since $\gamma^{Nash}>\gamma^{Coord}$ while the remaining parameters are positive. ■

H Numerical solution

In order to solve the model numerically we use the algorithm put forward by Soderlind (1999). The equilibrium conditions are cast in the following form:

$$\begin{bmatrix}x_{1t+1}\\E_tx_{2t+1}\end{bmatrix}=A\begin{bmatrix}x_{1t}\\x_{2t}\end{bmatrix}+Bu_t+\begin{bmatrix}\varepsilon_{t+1}\\\mathbf{0}_{n_2\times1}\end{bmatrix}. \tag{H.42}$$

In the expression above x_{1t} are predetermined variables and x_{2t} are non-predetermined variables. The policy instrument is denoted by u_t and ε_{t+1} are innovations to x_{1t}. Under discretion the policy problem is given in general terms by the following expression:

$$x_t'Vx_t + v_t = \min_{u_t}\left[x_t'Qx_t + 2x_t'Uu_t + u_t'Ru_t + \beta E_t\{x_{1t+1}'Vx_{1t+1} + v_{t+1}\}\right] \tag{H.43}$$
$$\text{s.t.} \quad E_t x_{2t+1} = Cx_{1t+1}, \text{ Eq. (H.42), and } x_{1t} \text{ given,}$$

where $x_t'Vx_t$ is the value function (quadratic in the state variables), $x_t'Qx_t+2x_t'Uu_t+u_t'Ru_t$ is the period loss function and $x_{2t} = Cx_{1t}$ maps predetermined variables into non-predetermined variables.

Equations (4), (6) and (7) can be cast into this setup by rewriting them as follows:

$$E_t\{\pi_{t+1}^i\} = \frac{1}{\beta}\left(1+\kappa(1-\gamma)\right)\pi_t^i + \frac{\gamma}{\beta}\left(\frac{\lambda}{1-\gamma} - \kappa\right)\hat{g}_t^i - \frac{1}{\beta}\kappa(1-\gamma)(\hat{c}_t^* + \pi_t^* + s_{t-1}^i)$$
$$s_t^i = \pi_t^* - \pi_t^i + s_{t-1}^i.$$

The vectors in (H.42) are thus given by

$$x_{1t} = \begin{bmatrix} s_{t-1}^i \\ \pi_t^* \\ \hat{c}_t^* \end{bmatrix}, \; x_{2t} = \begin{bmatrix} \pi_t^i \end{bmatrix}, \; u_t = \hat{g}_t^i, \; \varepsilon_t = \begin{bmatrix} 0 \\ \pi_t^* \\ \hat{c}_t^* \\ 0 \end{bmatrix}, \; x_t = \begin{bmatrix} s_{t-1}^i \\ \pi_t^* \\ \hat{c}_t^* \\ \pi_t^i \end{bmatrix},$$

where x_t is introduced for notational convenience. Matrices A and B are given by

$$A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{\beta}\kappa(1-\gamma) & -\frac{1}{\beta}\kappa(1-\gamma) & -\frac{1}{\beta}\kappa(1-\gamma) & \frac{1}{\beta}\left(1+\kappa(1-\gamma)\right) \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\gamma}{\beta}\left(\frac{\lambda}{1-\gamma} - \kappa\right) \end{bmatrix}.$$

We multiply the period loss function by 2 and rearrange it to

$$\frac{\varepsilon}{\lambda}(\pi_t^i)^2 + \left((1+\varphi) + \frac{\gamma^{Coord}}{1-\gamma^{Coord}}\right)(\hat{y}_t^i)^2 - 2\frac{\gamma^{Coord}}{1-\gamma^{Coord}}\hat{y}_t^i\hat{g}_t^i + \frac{\gamma^{Coord}}{1-\gamma^{Coord}}(\hat{g}_t^i)^2.$$

This can be written as

$$\begin{bmatrix} \pi_t^i & \hat{y}_t^i & \hat{g}_t^i \end{bmatrix} \begin{bmatrix} \frac{\varepsilon}{\lambda} & 0 & 0 \\ 0 & (1+\varphi) + \frac{\gamma^{Coord}}{1-\gamma^{Coord}} & -\frac{\gamma^{Coord}}{1-\gamma^{Coord}} \\ 0 & -\frac{\gamma^{Coord}}{1-\gamma^{Coord}} & \frac{\gamma^{Coord}}{1-\gamma^{Coord}} \end{bmatrix} \begin{bmatrix} \pi_t^i \\ \hat{y}_t^i \\ \hat{g}_t^i \end{bmatrix} = \begin{bmatrix} \pi_t^i & \hat{y}_t^i & \hat{g}_t^i \end{bmatrix} W \begin{bmatrix} \pi_t^i \\ \hat{y}_t^i \\ \hat{g}_t^i \end{bmatrix}.$$

We define the auxiliary matrix K:

$$\begin{bmatrix}\pi_t^i\\ \hat{y}_t^i\\ \hat{g}_t^i\end{bmatrix}=\begin{bmatrix}0&0&0&1&0\\(1-\gamma)&(1-\gamma)&(1-\gamma)&-(1-\gamma)&\gamma\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}s_{t-1}^i\\ \pi_t^*\\ \hat{c}_t^*\\ \pi_t^i\\ \hat{g}_t^i\end{bmatrix}$$

$$=K\begin{bmatrix}x_t\\ u_t\end{bmatrix},$$

where we make use of

$$\hat{y}_t^i=\gamma\hat{g}_t^i+(1-\gamma)\hat{c}_t^*+(1-\gamma)(\pi_t^*-\pi_t^i+s_{t-1}^i).$$

The loss function can therefore be written as

$$\begin{bmatrix}\pi_t^i & \hat{y}_t^i & \hat{g}_t^i\end{bmatrix}\begin{bmatrix}\frac{\varepsilon}{\lambda} & 0 & 0\\ 0 & (1+\varphi)+\frac{\gamma^{Coord}}{1-\gamma^{Coord}} & -\frac{\gamma^{Coord}}{1-\gamma^{Coord}}\\ 0 & -\frac{\gamma^{Coord}}{1-\gamma^{Coord}} & \frac{\gamma^{Coord}}{1-\gamma^{Coord}}\end{bmatrix}\begin{bmatrix}\pi_t^i\\ \hat{y}_t^i\\ \hat{g}_t^i\end{bmatrix}=\begin{bmatrix}x_t' & u_t'\end{bmatrix}K'WK\begin{bmatrix}x_t\\ u_t\end{bmatrix}.$$

Therefore, the matrices in (H.43) are given by

$$\begin{bmatrix}Q & U\\ U' & R\end{bmatrix}=K'WK.$$

The solution to (H.43) gives the policy rule (see Soderlind, 1999):

$$u_t=-Fx_{1t}.$$

Put differently

$$\hat{g}_t^i=-f_1s_{t-1}^i-f_2\pi_t^*-f_3\hat{c}_t^*.$$

In a symmetric equilibrium the terms of trade are zero and the equilibrium is determined at the union level, see Appendix F. The equilibrium conditions in the ELB scenario are:

$$\begin{aligned}
\hat{g}_L^{*,Nash}&=-f_2\pi_L^{*,Nash}-f_3\hat{c}_L^{*,Nash}\\
\pi_L^{*,Nash}&=\frac{1}{1-\beta\mu}\kappa(\hat{y}_L^{*,Nash}-\frac{\bar{\sigma}\gamma}{\bar{\sigma}+\varphi}\hat{g}_L^{*,Nash})\\
\hat{y}_L^{*,Nash}&=\frac{(1-\gamma)(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-(1-\gamma)\mu\kappa}r_L+\frac{(1-\mu)(1-\beta\mu)\gamma-(1-\gamma)\mu\kappa\frac{\gamma\bar{\sigma}}{\bar{\sigma}+\varphi}}{(1-\mu)(1-\beta\mu)-(1-\gamma)\mu\kappa}\hat{g}_L^{*,Nash}\\
\hat{y}_L^{*,Nash}&=\gamma\hat{g}_L^{*,Nash}+(1-\gamma)\hat{c}_L^{*,Nash}.
\end{aligned}$$

Given the numerical solution for f_2 and f_3 we solve the above system for $\hat{g}_L^{*,Nash}$.

I Consumption equivalent

We compute the consumption equivalent as the (constant) value that has to be given to the household in the absence of coordination in order to make her in expected terms as well off as under coordination. Below we are more explicit about the exact expression for the consumption equivalent. We start by computing expected lifetime utility assuming that the economy is initially at the ELB in period 0. With probability μ the ELB remains binding and with probability $1-\mu$ the economy reverts permanently back to steady state. Denote by U_L the utility of the household in a given period at the ELB and by U the utility of the household in a given period in steady state. The following chart—in which the arrows depict transition probabilities—illustrates how to compute expected discounted lifetime utility $\tilde{U}_0 = E_t \sum_{t=0}^{\infty} \beta^t U_t$:

$$
\begin{array}{r|lllll}
 & \text{Expected discounted utility} & & & & \\
\hline
\text{Period 0:} & U_L & & & & \\
 & \downarrow_{\mu} & \searrow_{1-\mu} & & & \\
\text{Period 1:} & \mu\beta U_L & (1-\mu)\beta U & & & \\
 & \downarrow_{\mu} & \searrow_{1-\mu} & \searrow_{1} & & \\
\text{Period 2:} & (\mu\beta)^2 U_L & (1-\mu)\mu\beta^2 U & (1-\mu)\beta^2 U & & \\
 & \downarrow_{\mu} & \searrow_{1-\mu} & \searrow_{1} & \searrow_{1} & \\
\text{Period 3:} & (\mu\beta)^3 U_L & (1-\mu)\mu^2\beta^3 U & (1-\mu)\mu\beta^3 U & (1-\mu)\beta^3 U & \\
\vdots & \underbrace{\quad\vdots\quad}_{U_L \sum_t (\mu\beta)^t} & \underbrace{\quad\vdots\quad}_{(1-\mu)\beta U \sum_t (\mu\beta)^t} & \underbrace{\quad\vdots\quad}_{(1-\mu)\beta^2 U \sum_t (\mu\beta)^t} & \underbrace{\quad\vdots\quad}_{(1-\mu)\beta^3 U \sum_t (\mu\beta)^t} & \ddots \\
 & & & & & \dots
\end{array}
$$

Therefore, expected discounted lifetime utility can be written as:

$$
\tilde{U}_0 = U_L \sum_{t=0}^{\infty} (\mu\beta)^t + (1-\mu)\beta U \left[\sum_{t=0}^{\infty} (\mu\beta)^t \right] \left[\sum_{t=0}^{\infty} \beta^t \right], \tag{I.44}
$$

where

$$
U_L = (1-\chi)\log C_L + \chi \log G_L - \frac{(N_L)^{1+\varphi}}{1+\varphi}
$$
$$
U = (1-\chi)\log C + \chi \log G - \frac{N^{1+\varphi}}{1+\varphi}.
$$

Since $\mu\beta < 1$ the geometric series in (I.44) converges to:

$$
\tilde{U}_0 = \left(\frac{1}{1-\mu\beta} \right) U_L + \left(\frac{1-\mu}{1-\mu\beta} \right) \left(\frac{\beta}{1-\beta} \right) U.
$$

The consumption equivalent is a constant compensation that makes the household in the absence of coordination as well off as under coordination. More specifically, the household receives a compensation ζ such that the following equality holds

$$\tilde{U}_0^{Coord} = \tilde{U}_0^{Nash}(\zeta), \tag{I.45}$$

where

$$\tilde{U}_0^{Nash}(\zeta) = \left(\frac{1}{1-\mu\beta}\right) U_L^{Nash}(\zeta) + \left(\frac{1-\mu}{1-\mu\beta}\right)\left(\frac{\beta}{1-\beta}\right) U^{Nash}(\zeta),$$

with

$$\begin{aligned}
U_L^{Nash}(\zeta) &= (1-\chi)\log(C_L^{Nash}(1+\zeta)) + \chi \log G_L^{Nash} - \frac{\left(N_L^{Nash}\right)^{1+\varphi}}{1+\varphi}, \\
U^{Nash}(\zeta) &= (1-\chi)\log(C^{Nash}(1+\zeta)) + \chi \log G^{Nash} - \frac{\left(N^{Nash}\right)^{1+\varphi}}{1+\varphi}.
\end{aligned}$$

Inserting in (I.45) yields:

$$\begin{aligned}
\tilde{U}_0^{Coord} = & \frac{1}{1-\mu\beta}\left[(1-\chi)\log(C_L^{Nash}(1+\zeta)) + \chi \log G_L^{Nash} - \frac{\left(N_L^{Nash}\right)^{1+\varphi}}{1+\varphi}\right] \\
& + \left(\frac{1-\mu}{1-\mu\beta}\right)\left(\frac{\beta}{1-\beta}\right)\left[(1-\chi)\log(C^{Nash}(1+\zeta)) + \chi \log G^{Nash} - \frac{\left(N^{Nash}\right)^{1+\varphi}}{1+\varphi}\right].
\end{aligned}$$

Rearranging further

$$\begin{aligned}
&(1-\mu\beta)\,\tilde{U}_0^{Coord} - U_L^{Nash}(0) - (1-\mu)\left(\frac{\beta}{1-\beta}\right) U^{Nash}(0) \\
&= (1-\chi)\log(1+\zeta)\left[1 + (1-\mu)\left(\frac{\beta}{1-\beta}\right)\right].
\end{aligned}$$

The above equation can be rearranged to:

$$\zeta = \exp\left\{\frac{1}{1-\chi}\left[1 + (1-\mu)\left(\frac{\beta}{1-\beta}\right)\right]^{-1}\left((1-\mu\beta)\,\tilde{U}_0^{Coord} - U_L^{Nash}(0) - (1-\mu)\left(\frac{\beta}{1-\beta}\right) U^{Nash}(0)\right)\right\} - 1.$$

We back out G_L and Y_L from our numerical solution to equations (F.36)-(F.38) making use of $Y = N$ (which holds in a first order approximation). We use the non-linearized version of (5) to compute C_L. The steady state values are obtained from the solution to the social planner problems.