Discussion of "Mixed-Frequency Large-Scale Factor Models"

by M. Rubin, E. Andreou, P. Gagliardini and E. Ghysels

Thomas Götz

Maastricht University

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Outline

- Other factor models
- Paper's contribution
- Open issues

Other MF factor models I

- Mariano and Murasawa (2003): Single latent factor model (quarterly GDP + 4 monthly series)

$$\begin{pmatrix} y_{1t}^* \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \mu_{1t}^* \\ \mu_{2t} \end{pmatrix} + \beta f_t + u_t,$$

where f_t is scalar (latent) factor with AR(p) structure and u_t are idiosyncratic AR(q) shocks

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- Nunes (2005): Builds a monthly coincident index of economic activity including monthly series (nowcasting) and GDP

$$y_{i,t} = \beta_i + \gamma_i C_t + u_{i,t}$$

- $\Rightarrow$ State-space representation and Kalman filter both times

Other MF factor models II

- Aruoba, Diebold and Scotti (2009): Tracking and forecasting real activity, so single-factor model, i.e., x_t denoting underlying business conditions at day t is scalar

$$y_t^i = c_i + \beta_i x_t + \sum_{j=1}^{k} \delta_{ij} wjt + \sum_{j=1}^{n} \gamma_{i,j} y_{t-iD_i}^i + u_t^i,$$

where $D_i > 1$ is linked to the observed frequency of y^i

- y_t^i - daily time scale; Most variables actually observed at lower frequency $\Rightarrow$ Again missing values and state-space representation
- All factor models thus far restricted to small set of series

Other MF factor models III

- Marcellino and Schumacher (2010): Introduces Factor MIDAS for now-/forecasting LF series using a large set of HF indicators

$$y_{t_q+h_q} = \beta_0 + \beta_1 b(L_m, \theta)\hat{f}^{(3)}_{t_m+h_m} + \epsilon_{t_m+h_m}$$

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- Unbalanced dataset (real-time forecast application): Complete representation in state-space form (Doz et al., 2006, or Giannone et al., 2008)
- Related to the DFM of Doz et al. (2006) extended by Bańbura and Rünstler (2011)
- Not restricted to single-factor model only

Contribution

- This paper proposes a large-scale mixed-frequency factor model á la Bai and Ng (2002), Bai (2003), Bai and Ng (2006) allowing for...
  - ...large panels of observable HF and LF data
  - ...latent HF and LF factors
- An iterative estimation procedure using PCA is introduced and shown to lead to consistent estimators
- Method is applied to analyzing the effect of a HF (quarterly) Industrial Production factor on GDP growth of non-IP sectors in the economy (LF; annually)
  - Examine whether common factors reflect propagation of sectoral shocks between service sectors and manufacturing

What I like about the paper

- Fills an important gap in the (mixed-frequency) literature
- Cleverly combines the MF-VAR in Ghysels (2012) with the large-scale factor models in Bai and Ng (2002) etc.
- Very interesting, elaborate and valuable empirical application emphasizing the practical use of the method, especially on the explanatory power of LF and HF aggregate, technological shocks
- Well-written and interesting paper to read

Open Issues

- The underlying DGP
- The number of factors
- Minor issues

Is the world mixed-frequency?

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  - HF-DGP $\Rightarrow$ Deistler (2012), Miller (2012), the entire state-space branch, and so on
  - MF-DGP $\Rightarrow$ Andreou et al. (2010), Götz et al. (2013), your paper?
    - ★ Foroni and Marcellino (2013): Decision making of economic agents vs. true underlying sampling frequency of the data
  - "Both" $\Rightarrow$ Ghysels (2012), Ghysels et al. (2013), Götz et al. (2014)

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- As shown above, the underlying state of the economy is usually assumed to be evolving at some high frequency (e.g., daily in Aruoba et al., 2009)
- Of course, you can set K_L equal to zero. However, linearization and consistency results seem to rely on a DGP favouring your model:

  "Simulations simulated from a DGP calibrated on the empirical application [...]"

HF DGP?

- Put your model to the test: Use a common high-frequency DGP

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- Put your model to the test: Use a common high-frequency DGP
- Ghysels et al. (2013) start from a high-frequency VAR and see how Granger causality is recovered in a MF-VAR and LF-VAR

$$\begin{pmatrix} x_{1,t} \\ x_{2,t} \\ y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \Lambda & 0 & \Delta_{(1)} & 0 \\ 0 & \Lambda & 0 & \Delta_{(2)} \\ \Omega_{(1)} & 0 & B & 0 \\ 0 & \Omega_{(2)} & 0 & B \end{pmatrix} \begin{pmatrix} f_{1,t} \\ f_{2,t} \\ g_{1,t} \\ g_{2,t} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ u_{1,t} \\ u_{2,t} \end{pmatrix}$$

- In a manner similar to Ghysels (2012) (or Ghysels et al., 2013)), you can set up the factor dynamics
- Set $K_H^x = K_H^y = 1$ say

HF DGP?

- Perform temporal aggregation first on y (average- or skip-sampling) to get the MF-model (1) and subsequently on x to get the LF-model (2)
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  - the estimated loading
  - the parameter estimates governing the factor dynamics
  - the number of factors (see below)

  are recovered
- You may vary K_H^x and K_H^y
- One may suspect: You recover more/better in scenario (1) than (2), which would back up the approach you develop

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- You only check the step from (1) to (2) as to how the number of factors is affected (X_{LF})

The number of factors I

- The amount of factors is undoubtedly crucial when performing the iterative estimation procedure

$$\hat{G}^{(p-1)} = (\tilde{g}_1, \ldots, \tilde{g}_T)'$$

  is the $(T \times K_L)$ matrix of estimated LF factors
- Likewise for the HF factors $\hat{F}^{*(p)}$

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- Likewise for the HF factors $\hat{F}^{*(p)}$
- What if K_H and K_L are mis-specified (too many or too few factors)?
  - What is the effect on the estimation procedure (maybe no problem, because of orthogonality), on the estimation of the factor dynamics (May be an important issue here), on the entire analysis?

The number of factors II

- When estimating the factor dynamics, there seem to be several kinds of uncertainty
  (i) Around the estimation of the loadings and factors
  (ii) Around the estimates of the factor dynamics
  (iii) Around the number of factors $\Rightarrow$ How does this uncertainty affect the others?
- When estimating standard errors of the parameter estimates in the structural MF-VAR for the factors, do you take the uncertainty surrounding (i) and (iii) into account?

The number of factors III

TABLE 1 BIS: Estimated number of factors (HF data: IP indexes, LF: non-IP real **GROSS OUTPUT**)

IC_{p1}: growth rates of indexes

$[Y\ X_1]$	$[Y\ X_2]$	$[Y\ X_3]$	$[Y\ X_4]$	$[Y\ X_{1:4}]$	$[Y\ X_{LF}]$	$[X_{LF}]$	$[X_{HF}]$	$[Y]$
1	1	2	2	2	2	2	1	15

IC_{p2}: growth rates of indexes

$[Y\ X_1]$	$[Y\ X_2]$	$[Y\ X_3]$	$[Y\ X_4]$	$[Y\ X_{1:4}]$	$[Y\ X_{LF}]$	$[X_{LF}]$	$[X_{HF}]$	$[Y]$
1	1	2	2	2	2	1	1	2

IC_{p1}: **innovations to sectoral productivity** (ε_t in Foerster, Sarte, and Watson (2011))

$[\varepsilon_Y\ \varepsilon_{X1}]$	$[\varepsilon_Y\ \varepsilon_{X2}]$	$[\varepsilon_Y\ \varepsilon_{X3}]$	$[\varepsilon_Y\ \varepsilon_{X4}]$	$[\varepsilon_Y\ \varepsilon_{X1:4}]$	$[\varepsilon_Y\ \varepsilon_{X,LF}]$	$[\varepsilon_{X,LF}]$	$[\varepsilon_{X,HF}]$	$[\varepsilon_Y]$
1	1	1	1	1	2	3	1	15

IC_{p2}: **innovations to sectoral productivity** (ε_t in Foerster, Sarte, and Watson (2011))

$[\varepsilon_Y\ \varepsilon_{X,1}]$	$[\varepsilon_Y\ \varepsilon_{X,2}]$	$[\varepsilon_Y\ \varepsilon_{X,3}]$	$[\varepsilon_Y\ \varepsilon_{X,4}]$	$[\varepsilon_Y\ \varepsilon_{X,1:4}]$	$[\varepsilon_Y\ \varepsilon_{X,LF}]$	$[\varepsilon_{X,LF}]$	$[\varepsilon_{X,HF}]$	$[\varepsilon_Y]$
1	1	1	1	1	1	1	1	1

The number of factors IV

- "In our model, the number of factors is $K_L + K_H$ for panels $[Y\ X_i]$, $i = 1, 2, 3, 4$, $K_L + 4K_H$ for panel $[Y\ X_{1:4}]$ and $K_L + K_H$ for panel $[Y\ X_{LF}]$"

The number of factors IV

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- How can you from the table above deduct that $K_H = K_L = 1$?
  - According to panel $[Y\ X_{1:4}]$, you should get 5 factors
  - Clarify what K_H means $\Rightarrow$ Each HF ("fast") factor should correspond to m (MF literature) LF ("slow") factors, or not?
- How comes you get 15 factors among Y only?

The number of factors V

- How well do the information criteria by Bai and Ng (2002) capture the amount of factors?
- Andreou et al. (2013) $\Rightarrow$ All criteria select the maximum number of factors
- In relation to the previous point, you may check how
  - the number of factors selected by the various criteria changes depending on the DGP
  - the composition of the factors varies in response to the DGP
  - how the factors corresponding to different DGPs relate to each other (e.g., the effect of a number HF factors may get captured by one LF factor when going from (1) to (2))

The Empirical Analysis

- Are the data seasonally adjusted?
  - Shouldn't we expect, e.g., a summer factor?
- What if the data are non-stationary?
  - Factors are linear combinations of first differences $\Rightarrow$ Potentially neglecting crucial long-run relationships
  - What are the effects on the empirical analysis with components of GDP?
- "Study the effect of the common IP factor on the growth of the other sectors of the U.S. economy"
  - Isn't it enough to know that the share of the former falls in order to deduce an increase in the latter?

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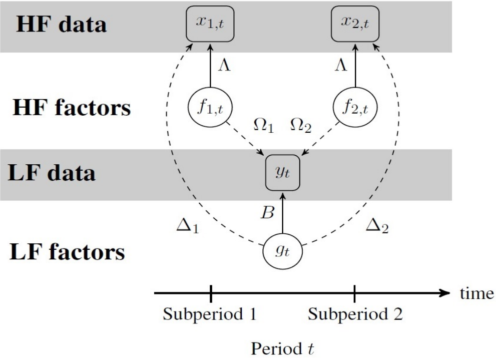
- Ω_2: x appears after y?



Potential future work

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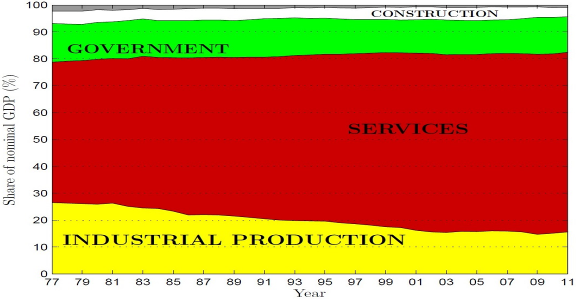

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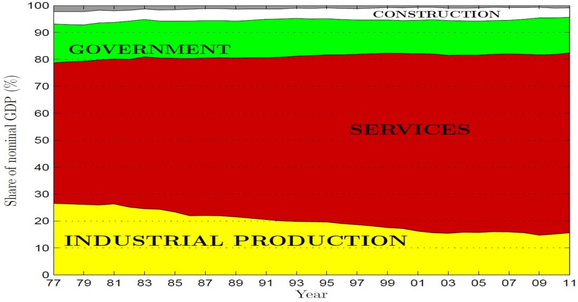

Do they load on the different sectors of the economy?

- Also related to Andreou et al. (2013), you may introduce time-varying factor loadings to address instabilities during the sample